NCCA

An Introduction to Using Cognitively Challenging Tasks

In the context of mathematics, **a task is an activity which is designed to focus learners' attention on particular mathematical ideas** (Stein et al., 1996). An important consideration when planning mathematical tasks is the cognitive challenge that they pose for learners. As signalled in Chapter 6, **cognitively challenging tasks (CCTs) offer rich learning opportunities that should appropriately stretch and challenge children's conceptual understanding**.

Zone of proximal development (ZPD) is a term used to describe the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers."

(Vygotsky, 1978, p.86)

In designing CCTs, teachers should ensure that

✓ the task is within a learner's zone of proximal development (ZPD) –

- it is closely linked to children's current level of knowledge and understanding but at the same time 'just beyond' their cognitive reach;
- the level of difficulty is not too high;

✓ the solution pathway is not immediately obvious to the learner;

✓ the solution is not reduced to a set of steps and procedures.

These are indicative of important aspects of CCTs. Firstly, a task in itself is not cognitively challenging – rather, cognitive challenge arises if the task makes appropriate demands of the learner. The incorporation of an element of choice in tasks as well as the use of parallel tasks are some of the means of addressing the needs of a variety of learners. Both of these will be explored in this support material. Secondly, open-ended tasks (where there is a range of 'correct' solutions and/or a range of ways to achieve one or more solutions) are more likely to be cognitively challenging than closed tasks (i.e., where there is only one correct response or where there is a focus on one solution pathway). Some ways to design open-ended tasks are shown below.

Also considered in this support material is the consolidation of learning, the use of realistic tasks and the kind of classroom environment that supports cognitive challenge. It should be noted that while various kinds of tasks (open-ended, parallel, realistic) are treated in separate subsections, they are not necessarily discrete, e.g., parallel tasks might be both open-ended and realistic.

Consolidation of learning using cognitively challenging tasks

Sometimes children's grasp of a new mathematical idea is fragile in that they can use it only in a specific context, or when they are dealing with a particular type of task. This is why **consolidation, where learners can generalise their learning and use new knowledge in a flexible and fluid way, is important**. CCTs can be designed to facilitate this consolidation. This can be achieved through using similar tasks, e.g., where children make the same pattern using different objects, (see example below of Making Patterns) or if the second task builds on the knowledge gained in the first task, e.g., 'Sending Cards' and 'Nature Gifts'. Children can also consolidate new learning if they are given the opportunity to revisit artefacts generated during their initial work on a CCT (e.g., writings, drawings, photographs) and re-evaluate them after, say, a few weeks.

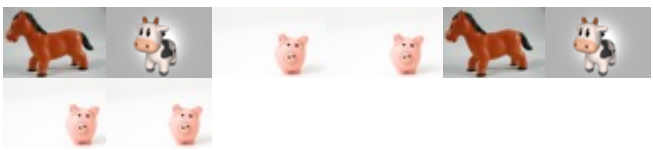
Making Patterns

A pair of children might create a pattern like this:

If they can confidently describe the pattern, say what comes next, what the rule is for their pattern, etc., they could be asked to recreate the same pattern rule with different objects.

'Can you use the farm animals to create a pattern with the same rule?'

The children would need to recognise they need three different items, one of which is duplicated. They may say they will use a horse instead of the circle, a cow instead of the square, a pig instead of the triangle, and create this instead:



Adapted from https://www.ncetm.org.uk/resources/52504

Sending Cards

Groups of children send cards to each other sometimes, perhaps at Christmas. It's a very nice idea because people like receiving cards.

Suppose there are three children, Molly, Jo and Al, and these three are really good friends. They decide to send cards to each other.

Molly sends two cards, one to Jo and one to Al.

Jo sends two cards, one to Molly and one to Al.

Al sends two cards, one to Jo and one to Molly.

- How many cards are sent altogether?
- Supposing there are 4 children who send cards to each other- how many cards will be sent?
- Explore the number of cards for 5, 6, and 7 children or even a class of 30 children.

Adapted from https://nrich.maths.org/56

Nature Gifts

During a week about the local environment at school, a group of three children give a present of 2 nature gifts (e.g., a plant seed, a shell, a leaf) to each other.

Mia gives four nature gifts, two to Jack and two to Ava.

Jack gives four nature gifts, two to Mia and two to Ava.

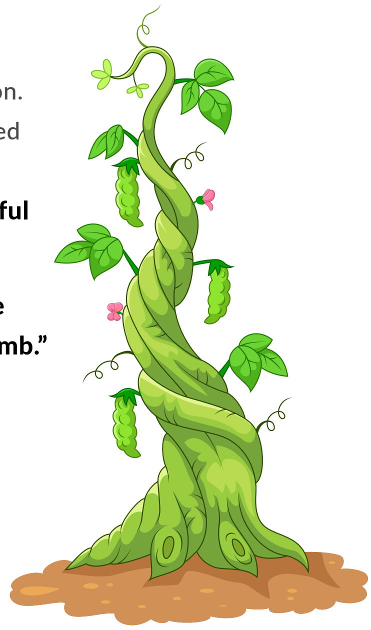
Ava gives four nature gifts, two to Jack and two to Mia.

- How many nature gifts are given altogether?
- Supposing there are 4 children who give nature gifts to each other- how many nature gifts are given altogether?
- Explore the number of nature gifts for 5, 6, and 7 children or even a class of 30 children.

Supporting engagement in CCTs

The use of 'realistic' problems is highly motivational for children. **This means that the task should be set in a context that helps learners to think in a 'real' way about the particular mathematical ideas involved. This does not mean that the context has necessarily to be 'real-world'. What is more important is that children can imagine or visualise the situation in a mathematical way**. The contexts can derive from mathematics itself, from all other subject areas, and from children's everyday lives. An example of a CCT arising from a story is shown below.

A CCT based arising from a story



After reading "Jack and the Beanstalk" aloud, the teacher introduced an estimation lesson. She reminded the children that Jack had received only a few beans for the cow. She asked the class,

"How many beans do you think Jack would have had if he'd received a whole handful instead of just a few?"

"Think about how many beans you think you can hold in a handful," she said. **"Trace one of your hands onto paper, cut it out, and write your estimate on the paper thumb."** Then do the actual count and compare your answers.

Children can decide whether they want to use to use large or small beans (e.g., kidney beans). If they select large beans they will quickly discover that a low number will be required to fill a handful. Some children might be drawn to the challenge of counting to, say, more than ten and thus opt for a smaller bean.

(Adapted from Burns, 1996)

A CCT arising from mathematics

BROKEN BUTTONS

'Broken Buttons' is a calculator activity that supports exploration of number operations. It can be adapted easily to suits the needs of various learners and class levels. Some examples of tasks are:

Suppose the '1' button on the calculator is broken. How would you display 19?

Suppose the '4' button on the calculator is broken. How would you solve 24 x 6?

Suppose the '4' and the '2' buttons on the calculator are broken. How would you solve 47 x 32?

Teachers can use their awareness of children's interests in devising CCTs. However, **it is crucial that the context allows children to visualise particular mathematical ideas**, e.g., a double-decker bus is a useful context for thinking about combinations of a numbers, a chocolate bar with eight squares can be useful for thinking about the relationship between halves, quarters and eighths, and beaded jewellery can support focus on patterns.

A suitable context can also derive from the problems posed by children as they engage in play activities or in 'wonderings', e.g.,

I wonder how tall I can make this tower of blocks before it falls ...

I wonder how many words are on the page ...

I wonder what the biggest number is ...

While teachers can capitalize on these 'teachable moments' with individual or small groups of children, they might also choose to use them as the basis of a mathematics lesson, if appropriate.

It is always possible that unexpected mathematical ideas will emerge during children's engagement with CCTs. The teacher can decide whether it is productive to capitalise on these 'contingency' moments in real time or to address them as appropriate at a later time.

The 'drill and practice' questions that are found in worksheets and textbooks can lead to children viewing the learning of mathematics as the acquisition and application of rules and procedures, and 'being good at mathematics' as following correctly a set of given rules. Children can end up feeling frustrated if they have not grasped the procedure, or bored if it is well within their reach. However, **worksheets and textbook activities can be adapted and used in a cognitively challenging manner**.

Children can be asked:

Why do you think that questions are ordered in the way they are?

Which is the most difficult for you? The easiest for you?

How are the questions the same? How are they different?

Which of the questions are most useful for you in your everyday life?

Select and complete five questions that would help your mathematics learning.

Number facts in table books can be treated in a similar manner.

The classroom environment

While a task might be carefully planned by a teacher with children's needs in mind, it is also important that the level of cognitive challenge remains when it's carried out in the classroom. A task can **remain cognitively challenging** throughout a lesson if

- the focus is on ways of thinking rather than on correct solutions;
- there is adequate time for children to complete the task;
- the teacher provides appropriate hints and continues to press for justification and explanation.

Factors that cause a reduction in cognitive challenge include a provision of a set of procedures, an emphasis on correct answers or a single solution method, or by providing children with an inappropriate amount of time (Stein et al., 1996).

Teachers can promote and maintain the cognitive challenge of tasks for the children through the classroom environment they create. For example, **it is important that children feel free to propose ideas without being afraid of making mistakes, and in turn, respect each other's ideas**. Sometimes it may be appropriate for children to make their contribution digitally, e.g., via a platform such as Mentimeter, followed by group or whole-class conversations on various contributions.

Another important aspect of CCTs is time for children to engage in 'possibility thinking' which means that they consider "What can I or we do with this?" (Craft and Chappell, 2016). Children need time and space to engage in this thinking independently, with other children and with the teacher, as appropriate to the task at hand. They also need access to a range of resources, and choices over how to engage (e.g., drawing, engaging in discussion, or choosing resources that they think are appropriate for the task at hand).

CCTs in an inclusive classroom

The identification of mathematical learning goals is a critical starting point for planning CCTs. In any class there is a range of different needs and the challenge for the teacher is to devise a CCT that responds both to this range of needs as well as to the mathematical learning goals. While it would not be possible or even advisable for a teacher to create a different task for each child in a class, there is a variety of ways that a teacher can facilitate an inclusive mathematics learning environment.

Small (2017) suggests that what is particularly important in meeting the needs of different learners is to give them **choice** in the way that they pursue the learning goal of the lesson. In this support material, various ways that choice can be integrated into tasks are exemplified.

Every child is capable of engaging in CCTs that meet their learning needs in mathematics. Through engagement with CCTs, learners are encouraged to connect different mathematical ideas; to persevere and take risks; and to devise solution strategies that make sense to them. Learning in this way is meaningful and robust. Furthermore, engagement in CCTs helps children to see that mathematics is not about following fixed procedures but rather that it is a creative subject, and one in which they have an important contribution to make.

Incorporation of choice

In general, children should be able to choose how they engage with a mathematical task and should have the opportunity to represent and express their understanding in a variety of ways.

Parallel tasks

Parallel tasks are sets of two or three tasks that are designed to meet the different needs of children but that get at the same mathematical idea. For example, it might be the case that some children in a class are not ready for the addition of two-digit numbers, and that working with addition of single-digit numbers is within their ZPD.

A possible set of parallel tasks is:

There are 35 children on the lower deck of a bus and 27 on the upper deck. How many children do you think there are in total?	There are 35 children on the lower deck of a bus and 23 on the upper deck. How many children do you think there are in total?	There are 5 children on the lower deck of a bus and 7 on the upper deck. How many children do you think there are in total?

In these examples, choice can be incorporated by encouraging children to select one task from the set of three, or to use their preferred way of approaching the task, such as with materials or inventing their own algorithm. These tasks are similar enough in context that all children can participate fully in a single follow-up discussion, e.g.,

- Did your solution surprise you? Why/Why not?
- Does your solution make sense to you? Why?
- How would you convince someone else that your solution is correct?
- How would your answer have changed if there was one extra person on the lower deck?

Open-ended tasks

An open-ended task is one where there is a range of 'correct' solutions and/or a range of ways to achieve one or more solutions. As mentioned above, open-ended tasks tend to be more cognitively challenging for all learners than closed tasks.

What two fractions add together to give ½? is an example of a **low threshold, high ceiling task**. Such tasks are those which are accessible to everyone at the start but also do not limit those who need further challenge – as described by the nrich team 'everyone can get started, and everyone can get stuck' (https://nrich.maths.org/10345).

Example 1: A range of correct solutions

What two fractions add together to give 1/2? This question has a range of solutions and children can enter it at their own level.

Some children might consider pairs of fractions with which they are familiar (e.g., ¼ + ¼; ⅛+ ⅜), others might focus on various pairs of fractions with common denominators (e.g., 1/6 + 2/6; 6/20 + 4/20) and yet others might give consideration to fractions reduced to their lowest terms (e.g., 1/6 + 1/3; 1/10 + 2/5).

Children can also choose different ways to represent (e.g., concrete materials, symbols) and express (pictorially, orally, in written form) their solutions.

Example 2: A range of ways to achieve one solution

If the mathematical focus of the task is the exchange of units for tens when combining numbers, a suitable task might be 35 + 27= ?

Here, the solution can be found and represented by learners in a variety of ways, e.g., with base ten materials, drawings, orally, with numerals.

	35 + 27
Luke:	35 + 20 = 55
	55 + 7 = 62
Emma:	30 + 20 = 50
	5 + 7 = 12
	50 + 12 = 62
Lily:	35 + 30 = 65
	65 – 3 = 62

A key aspect of children engaging in open-ended tasks is the follow-up discussions that take place either in a small group or whole-class setting. If the emphasis is placed on the generation of different ideas, all children can feel that they have something to contribute and, moreover, can learn from the ideas and strategies of their peers, e.g.,

What solution did you find to 35 + 27? Can you explain how you found it?

Which method do you think is most unusual?

Which, in your opinion, is a method that gives a solution quickly?

A closed task (i.e., where there is only one correct response or where there is a focus on one solution pathway) can be rendered open by 'turning it around'. An example of this is shown in the 'Turning a question around' example. Another way to create an open-ended task is found in the 'Asking for similarities and differences' example. Such an approach can be used for many different scenarios.

Creating open-ended tasks 1:

Turning a question around

(i) Identify a mathematical concept.

(ii) Think of a question that has one response.

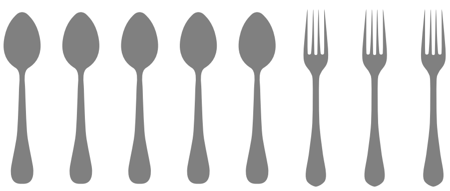
(iii) Make up a new question that includes (or addresses) the response.

Example 1

Mathematical concept: Subtraction can be represented as difference.

Question with one response:

How many more spoons than forks are there?



Open question:

Put out some spoons and forks. Make sure that you have more spoons than forks. Say or write how many more spoons than forks there are.

Example 2

Mathematical concept: A mean is a one-number summary of an entire distribution.

Question with one response:

Six children in a class recorded the number of minutes it took them to travel to school one day. This is what they found:

10, 20, 24, 12, 31, 5. What is the average number of minutes it took them to travel to school that day? (Ans.= 17 mins)

Open question:

Six children in a class recorded the number of minutes it took them to travel to school one day. They found the average number of minutes was 17. What might the six journey times have been? Explain why these journey times make sense to you.

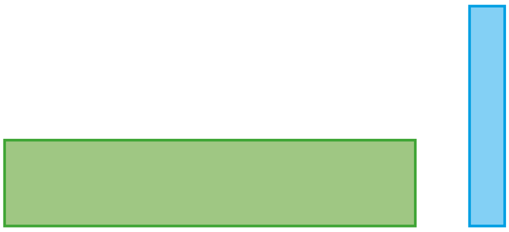
Creating open-ended tasks 2:

Asking for similarities and differences

Choose two items, e.g., two graphs, two numbers, two shapes, two measurements and ask how they are alike and how they are different.

Example 1

How are these shapes alike? How are they different?



Example 2

How are the number operations below alike? How are they different?

25 x 7

35 ÷ 7

Adapted from Small (2017)

Resources

As well as adapting existing tasks, CCTs can be also be found in websites and other published material, and modified as necessary for a particular context.

Useful Websites

- www.map.mathshell.org
- www.maths4all.ie
- www.mathsthroughstories.org
- www.nctm.org
- www.nrich.maths.org
- www.nzmaths.co.nz
- www.youcubed.org

Books

Burns, M. (1996). *50 Problem-solving Lessons: Grades 1-6*. Sausalito, CA: Math Solutions.

Small, M. (2017). *Good questions: Great ways to differentiate mathematics instruction in the standards-based classroom* (3rd ed.). New York, NY: Teachers College Press.

Smith, M., & Stein, M.K. (2011). *5 practices for orchestrating productive mathematics discussions.* Reston, VA: National Council of Teachers of Mathematics.

Sullivan, P., & Lilburn, P. (2004). *Open-ended maths activities* (2nd ed.). South Melbourne, Victoria: Oxford University Press.

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Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*(2), 455-488.

Vygotsky, L. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.