NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

Primary Mathematics Curriculum

Draft specification for consultation

March 2022

Contents

1. Introduction

Impetus for developments

The new Primary Mathematics Curriculum marks a significant landmark in the ongoing development of the curriculum for primary schools. Since the implementation of the 1999 Primary School Curriculum, reviews, evaluations and research have highlighted a number of strengths and challenges. Findings have indicated the need for a new Primary Mathematics Curriculum which

- is coherent in terms of aims, goals (relating to both processes and content), and pedagogy; particularly in acknowledging and building on the pedagogical emphases in *Aisteae: the Early Childhood Curriculum Framework* (NCCA, 2009).
- foregrounds the processes of communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, as central to the mathematical experiences of all children
- draws on current research in highlighting the principles and features of good mathematics pedagogy, including overarching pedagogical practices (See Chapter 6) and the ways in which they permeate learning activities
- supports teachers to design and develop rich and challenging mathematical tasks that are appropriate to their children's learning needs
- explicates the critical ideas in each of the content domains— Algebra, Data and Chance, Measures, Number, and Shape and Space
- presents learning paths which indicate shifts in children's mathematical learning and which serve as reference points for preparation for teaching, learning and assessment
- promotes the principles of equity and access for all children, ensuring that supports enable children with developmental challenges and those with exceptional talent, children in culturally diverse contexts and children in disadvantaged circumstances to experience rich and engaging mathematics.

The journey of development to date

It was initially planned that the Primary Mathematics Curriculum would be published in two parts, the first publication being the specification for junior infants to second class, followed by the specification for third to sixth class. The draft specification for junior infants to second class was subject to a robust consultation in 2017/2018, with a report of the consultation findings published (NCCA, 2018). In June 2018, a decision was made by the Minister for Education and Skills to publish the new Primary Mathematics Curriculum as a full specification from junior infants to sixth class. This decision was made following consideration of feedback from the system with regards to the implementation of the Primary Language Curriculum/Curaclam Teanga na Bunscoile in two phases.

Since then, research and development work has continued on a full extended specification. A number of changes and enhancements have been made to the initial draft of the curriculum, in response to the findings from the consultation in 2017/2018. These include the repositioning of the Progression Continua to the Primary Mathematics Toolkit; the addition of a new chapter 'The Primary Mathematics Curriculum in Practice' (see Chapter 6); and the development of a set of mathematical concepts to support teachers' planning and preparation for working with Learning Outcomes. NCCA has also developed an initial suite of ten Support Materials to attend the curriculum specification for this second consultation.

Research base for developments

The research base for curriculum developments was constructed in two phases. In the first phase, a systematic review of the literature was conducted, concentrating on teaching and learning in respect of children aged three to eight years. This comprised an international audit of mathematics curriculum policy (Burke, 2014); Research Report 17 (Dunphy et al., 2014) which focused on definitions, theories, development and progression in primary mathematics; and Research Report 18 (Dooley et al., 2014) which looked at pedagogy and learning more specifically. A synthesis of this research base for curriculum development produced the background paper and brief for development of the new Primary Mathematics Curriculum (NCCA, 2016). Following the publication of the first draft specification of the Primary Mathematics Curriculum for junior infants to second class (NCCA, 2017), a comprehensive phase of consultation took place between October 2017 to March 2018, which provided an opportunity to conduct valuable primary research at first hand on the draft new curriculum. Empirical data collected during, and subsequently reported from, this consultation (NCCA, 2018) added strongly to the research base for developments.

The second phase of research reports served to complement the existing research base by focusing on the senior classes of primary school. A research addendum to Research Reports 17 and 18 was compiled (Dooley, 2019) which looked at broad teaching and learning considerations for children in the upper years of primary school. This was further supplemented with five short research papers which examined core mathematical concepts, skills and processes with which children engage across the five mathematical domains (Delaney, 2020; Leavy, 2020; Nic Mhuirí, 2020a, 2020b; Twohill, 2020).

Overview of the new Primary Mathematics Curriculum

Figure 1: Curriculum and Toolkit

The new Primary Mathematics Curriculum is accompanied by the Primary Mathematics Toolkit.

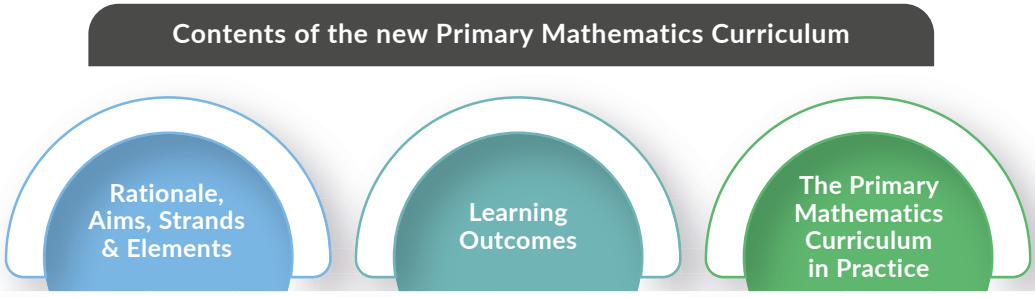


Figure 2: Overview of components of the Primary Mathematics Curriculum

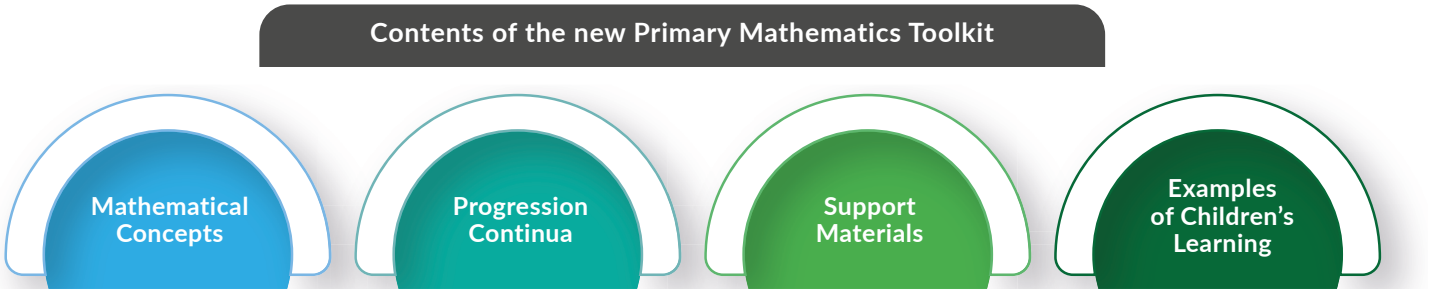


Figure 3: Overview of components of the Primary Mathematics Toolkit

Key components of the new Primary Mathematics Curriculum

Strands

The Strands represent the main areas or domains of learning in mathematics. The Primary Mathematics Curriculum has five Strands: Algebra, Data and Chance, Measures, Number, Shape and Space. The Strands are not discrete domains of learning; rather, it is important that teachers understand and plan for the interconnected nature of the Strands so as to support children to make sense of their learning.

Learning Outcome Labels

The strands are divided into Learning Outcome Labels, which give additional structure to the Curriculum. There are 15 Learning Outcome Labels in total. Each Learning Outcome Label contains a set of Learning Outcomes.

Elements

Elements describe essential mathematical learning in terms of the processes that are central to this learning. Children develop their mathematical proficiency through engaging with processes such as connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising. In the new Primary Mathematics Curriculum, for ease of reference, there are four Elements—Understanding and Connecting; Communicating; Reasoning; Applying and Problem Solving.

Learning Outcomes

Learning Outcomes describe the expected mathematical learning and development for children at the end of a two-year period, when due account is taken of individual abilities and varying circumstances. Learning Outcomes are an important reference point for planning and preparing for teaching and learning; and for assessment of mathematical learning in the classroom. It is intended that the Learning Outcomes help teachers to

- prepare for, teach and reflect on their use of appropriate methods for teaching and learning mathematical ideas
- focus the use of assessment to gather evidence of children's learning and understanding, thereby enabling teachers to adapt their teaching and respond appropriately to children's learning
- provide focused feedback to children and parents/ guardians.

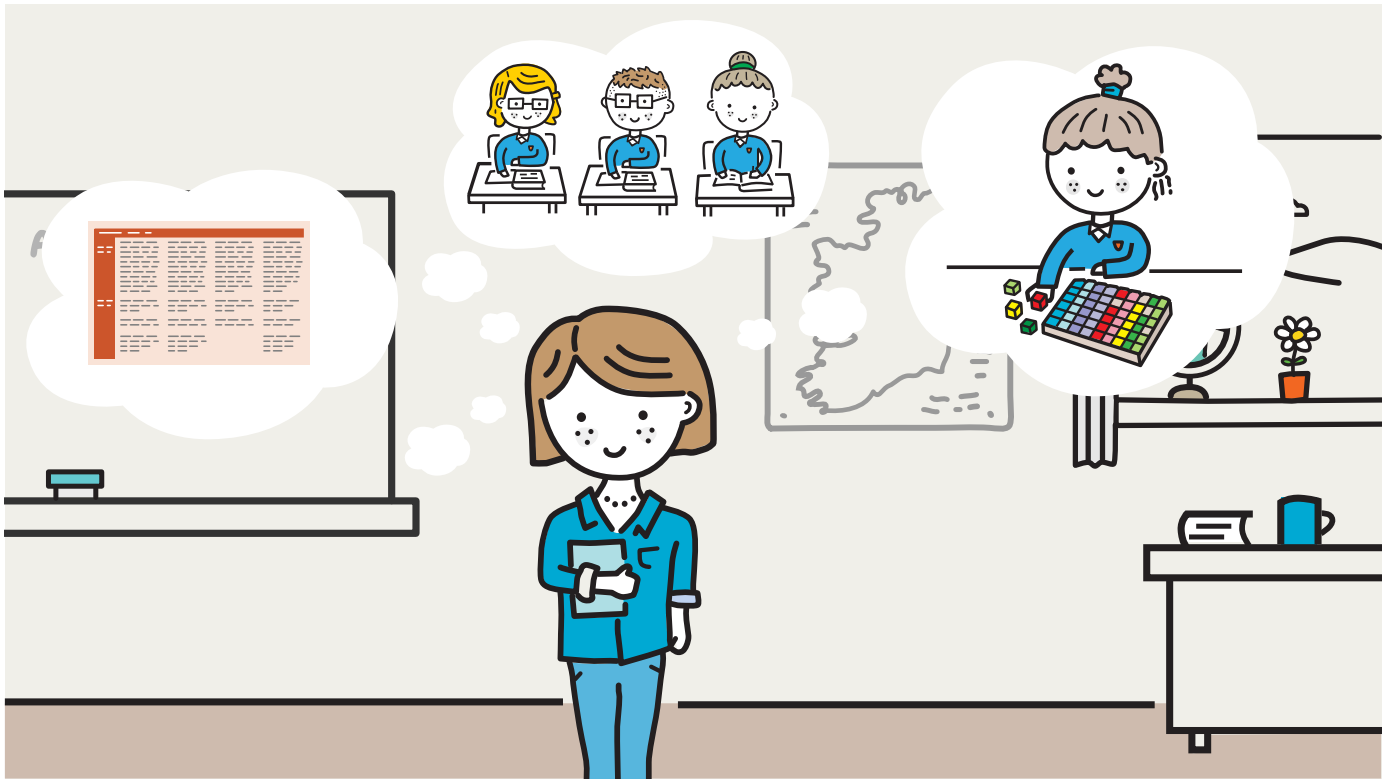


Figure 4: Using learning outcomes to build rich learning experiences in the classroom

Navigating the Curriculum

The opening sections of the Primary Mathematics Curriculum present the Rationale, Aims, Strands and Elements, and Learning Outcomes. The curriculum also provides guidance on the curriculum in practice, and the use of the Primary Mathematics Toolkit in planning and preparing for rich learning experiences. A glossary of terms and appendices are also provided.

In using the Curriculum, teachers should determine the Learning Outcome(s) that is(are) appropriate for the children in the classroom. This will help orientate planning and preparation for teaching, learning and assessment around key ideas in mathematics.

To support teachers to make decisions about the kind of learning experiences that will be most appropriate, the following supports are available

- Chapter 6 'The Primary Mathematics Curriculum in Practice' which outlines key pedagogical practices to develop mathematical proficiency
- The Primary Mathematics Toolkit. Supports include
    - Mathematical Concepts which describe essential ideas that underpin each of the Learning Outcomes across the four stages.
    - Progression Continua which suggest a trajectory of rich learning experiences that offer a practical support to progress children's thinking, skills and understanding on their mathematical learning journey.
    - Support Materials for teachers which offer subject matter-related and pedagogical content knowledge-related supports, resources and reference material.
    - Examples of Children's Learning which demonstrate what rich mathematical learning experiences may look like.[1]

Beyond the curriculum, there are a number of resources and opportunities that teachers may wish to avail of in supporting their enactment of the new Primary Mathematics Curriculum. These might include research articles, online resources, reflective and collaborative opportunities and professional development.

[1] Examples of Children's Learning will be made available to schools following the final publication of the specification.

2. Rationale

The rationale for the new Primary Mathematics Curriculum addresses the importance of mathematics in children's lives. It describes mathematics in the context of children's learning and development and what this means in terms of provision of education to children in the primary school.

Primary Mathematics

Mathematics is the study of the relationships, connections and patterns that surround us and is thus intrinsic to our concept of the world. Mathematics greatly enhances our capacity to understand and engage fully with the world around us. A child's mathematical journey begins from birth and the draft Primary Mathematics Curriculum acknowledges the learning journeys all children are on. From birth, children learn mathematics through their interactions and experiences at home, in early childhood settings and into primary and post-primary school.

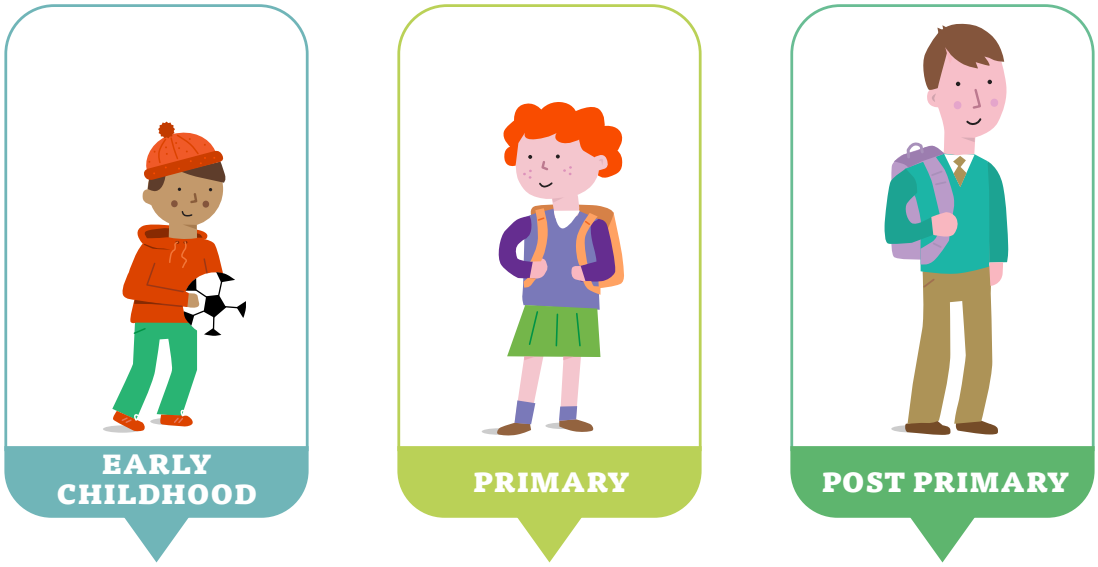


Figure 5: Learning journey in Mathematics

Every child is a mathematician

Every child has an innate, intuitive and instinctive sense of mathematics. Every child is capable of engaging with mathematical concepts and ideas from birth, and deepening and developing their learning over time.

Primary mathematics education should evoke children's innate ability to think and communicate mathematically, to solve problems and to make sense of the world using mathematics. Children should be encouraged to have a positive disposition to mathematics and to develop their mathematical understanding, language, communication skills, perseverance and resilience, interactions and expressions. Enabling children as mathematicians lays the foundations for children to become confident and life-long learners.

Mathematics is both a human and social phenomenon

Mathematics learning is dependent on social and cultural experiences as well as on children's educational experiences in school.

Primary mathematics education should provide children with learning experiences that give rise to mathematical thinking, such as modeling, thinking aloud and 'Maths Talk'. It should also provide opportunities for children to collaborate, communicate mathematical thinking, and express their understanding in multiple ways and in various contexts.

Mathematics is a tool that helps us to make sense of our world

Mathematics is used to think about, see and organise our everyday lives and the world.

Primary mathematics education should enable children to communicate and solve real world problems in mathematical terms. It is also essential to support children to develop the language of mathematics. Thus, mathematical thinking should be promoted and 'Math Talk' should be integral to the teaching and learning process.

Mathematics is beautiful and worthy of pursuit in its own right

Through playful, creative and engaging learning opportunities, children can experience the beauty and power of mathematics. It is important that children have the opportunity to engage with mathematics as a discipline in its own right and to explore its many intriguing aspects.

Primary mathematics education should foster a love of mathematics. It should provide children with the opportunity to explore, discover and refine their ideas. Children should also be supported to think critically and flexibly, and to be creative and innovative in their approach to learning mathematics.

Mathematics is everywhere and for everyone

Mathematics is a human activity that develops in response to everyday problems and interactions.

Primary mathematics education should provide children with opportunities to engage with deep, meaningful and challenging mathematics in educational settings, including social and familial settings. Such engagement will result in children co-constructing knowledge and skills as they interact and collaborate to solve complex and real problems.

3. Aims

The over-arching aim of the Primary Mathematics Curriculum is the development of mathematical proficiency. Mathematical proficiency encompasses conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition (see Figure 6 below). Critically, all five aspects are interwoven and interdependent. As children develop proficiency in one aspect, there are developments in other aspects too. Mathematical proficiency becomes progressively more developed in children as their mathematical experiences become increasingly sophisticated and as they are exposed to good pedagogy. The curriculum supports teaching, learning and assessment that is congruent with this aim.

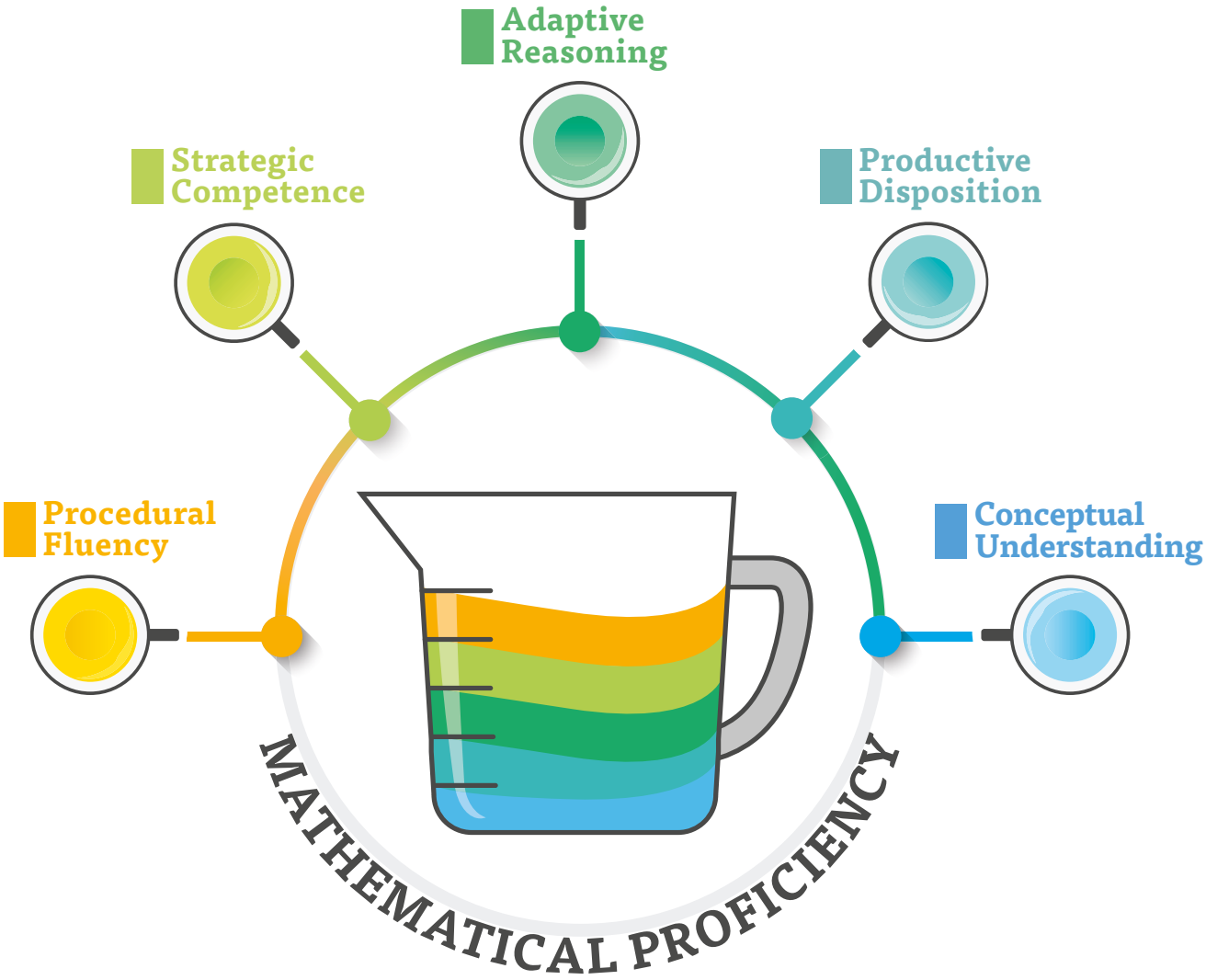


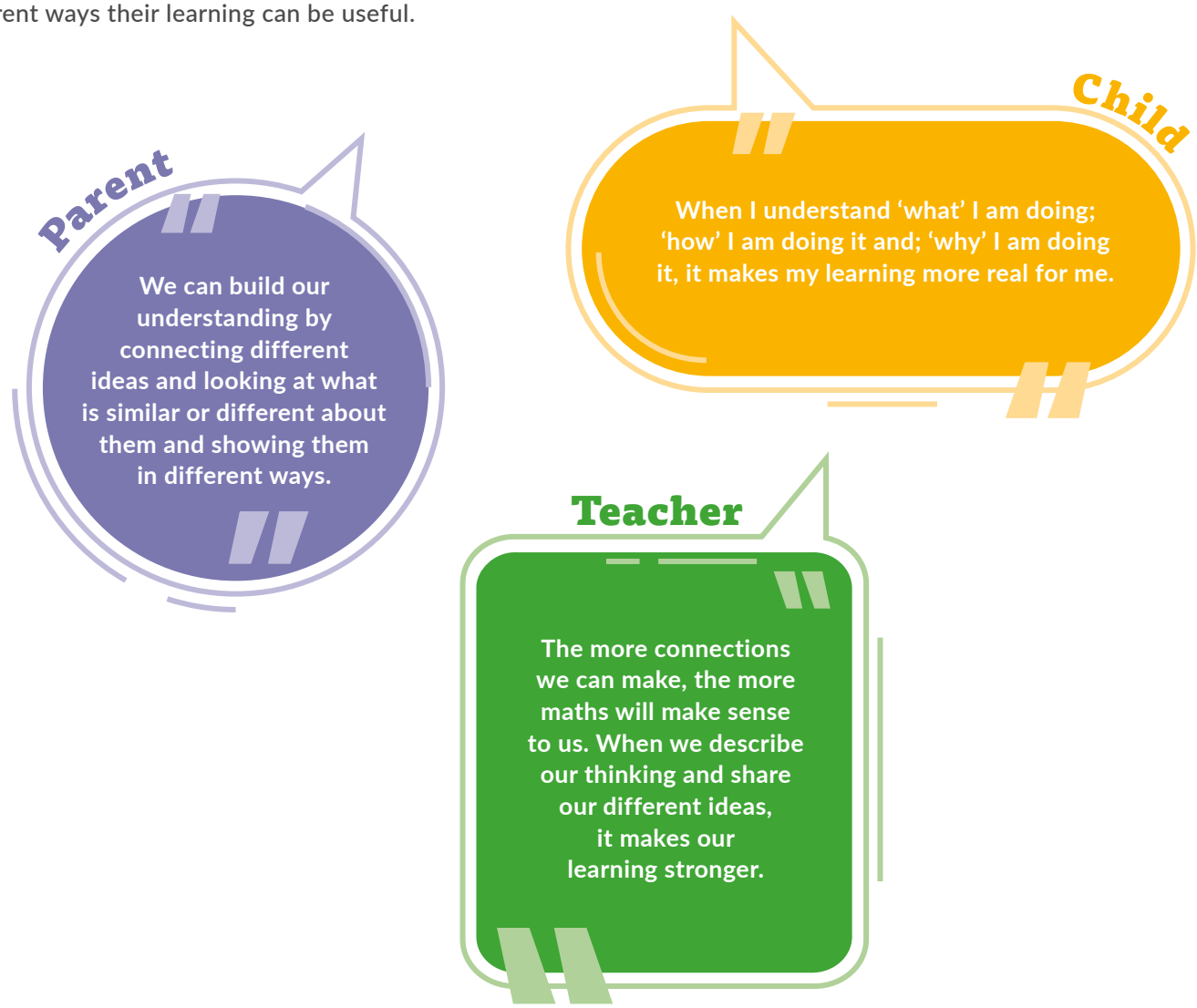
Figure 6: Five aspects of Mathematical Proficiency

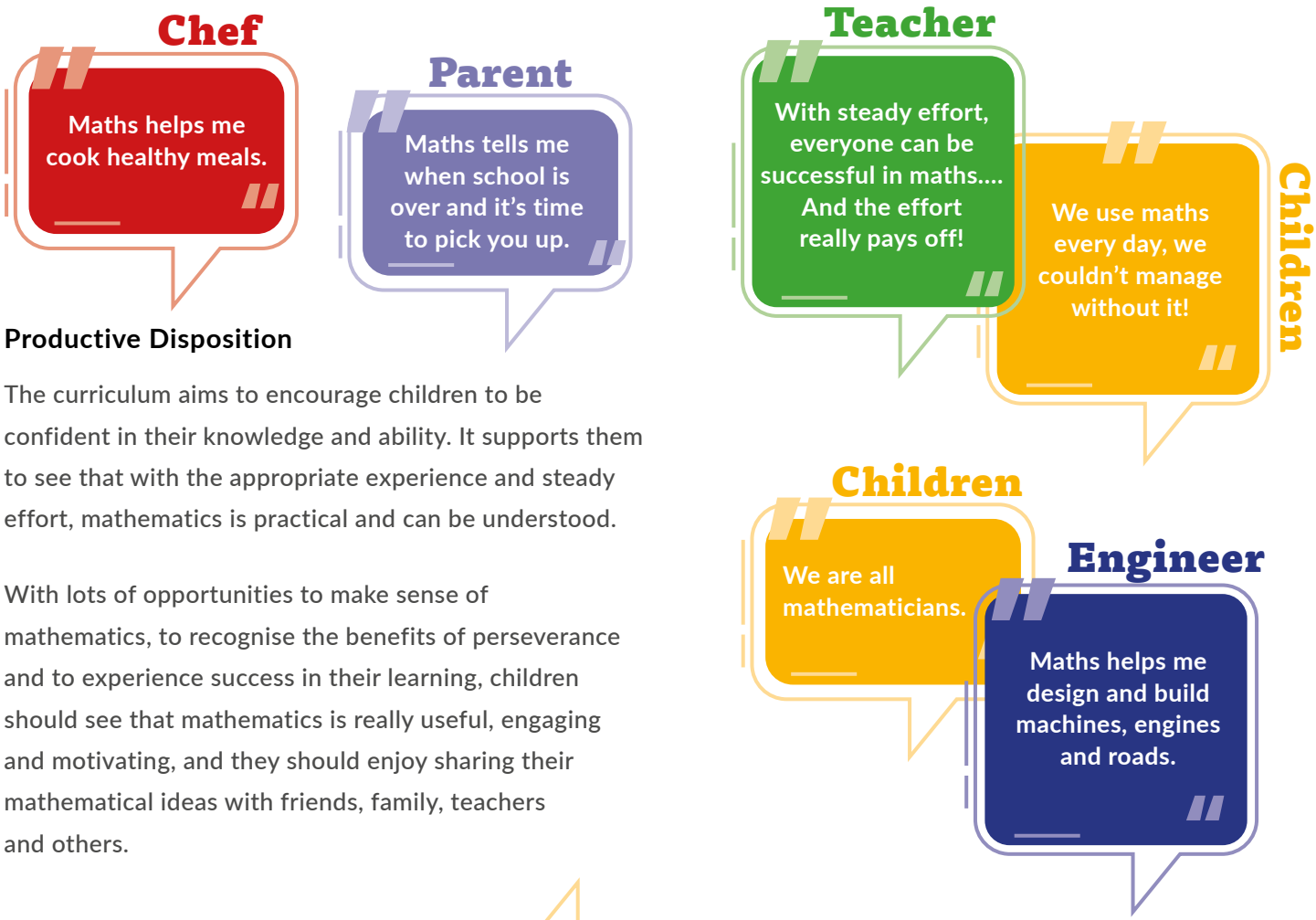
Conceptual Understanding

The curriculum aims to help children to understand why mathematics ideas are important and the different ways they can use and apply these ideas.

The curriculum aims to help children build on what they already know and to connect this with their new learning. This should make it easier for children to use their learning in different ways for different situations.

As well as supporting children to use mathematics in different ways, teachers should support children to represent how they understand mathematics in lots of different ways, such as using pictures or objects or by modelling, explaining and demonstrating it for their classmates. Having the opportunity to explore maths with others provides children with the opportunity to share and connect what they have learned, how they learned, and the different ways their learning can be useful.





Productive Disposition

The curriculum aims to encourage children to be confident in their knowledge and ability. It supports them to see that with the appropriate experience and steady effort, mathematics is practical and can be understood.

With lots of opportunities to make sense of mathematics, to recognise the benefits of perseverance and to experience success in their learning, children should see that mathematics is really useful, engaging and motivating, and they should enjoy sharing their mathematical ideas with friends, family, teachers and others.

Adaptive Reasoning

The curriculum aims to support children's capacity for logical thought, reflection, explanation, and justification. Through collaboration and a talk-friendly environment, teachers should enable children to reflect and navigate through the many concepts, solution methods, facts, and procedures they encounter so that they can see how mathematics fits together and makes sense.

Children should be encouraged to clarify and determine the legitimacy of their reasoning by discussing concepts and procedures; by representing problems, solutions and their understanding of mathematics in multiple ways; and by offering good reasons for the procedures and strategies they employ.

Procedural Fluency

The curriculum aims to provide children with opportunities to create their own informal strategies and to integrate new concepts and maths procedures as they build on these strategies. It aims to support children to justify the use of commonly used mathematical procedures and informal strategies, and through this, to strengthen their understanding and skills.

Teacher: This will help us solve lots of different problems that we will meet in the real world.

Parent: Different ways of doing maths might include recalling facts and definitions; devising clever plans to solve problems; choosing the best methods to help solve them; making good guesses and calculating answers.

Children should be encouraged to apply procedures accurately, efficiently and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognise when one strategy or procedure is more appropriate to apply than another.

Child: We need to know lots of different ways of doing maths; and how and when they can be most useful to us.

Strategic Competence

The curriculum aims to support children to become proficient problem solvers. Children should have the opportunity to formulate mathematical problems, represent them, and solve them in a variety of ways.

Teacher: In the real world, problems are rarely straightforward or specific. To be able to solve problems, and to make sense of math problems in school and at home, we need to practice, practice, practice.

As well as needing a range of solution strategies, children should be facilitated in generating problem models where they first understand the problem or situation and its key features, and then framing or representing the problem mathematically.

Children should detect mathematical relationships and devise mental representations of problems by building mental images of the essential components of the problem (variables and relations) using a number of tools such as numbers, concrete materials, manipulatives, symbols, words or graphics.

Teacher: Problem-solving is complex! But remember, the more practice we get to use our problem-solving skills, the better we will become in solving problems quickly and efficiently.

Children should also be encouraged to demonstrate flexibility throughout the problem-solving process and broaden their knowledge through solving meaningful, real-life problems and through creating or adjusting appropriate methods to fit the requirements of unfamiliar situations.

Child: That's right, we need to be able to pose a problem; make a plan; and then use familiar or new strategies that will help us to solve it.

4. Strands, Learning Outcome Labels and Elements

Curriculum Strands

The Curriculum Strands comprise five domains or content areas of primary mathematics: Algebra; Data and Chance; Measures; Number; Shape and Space.

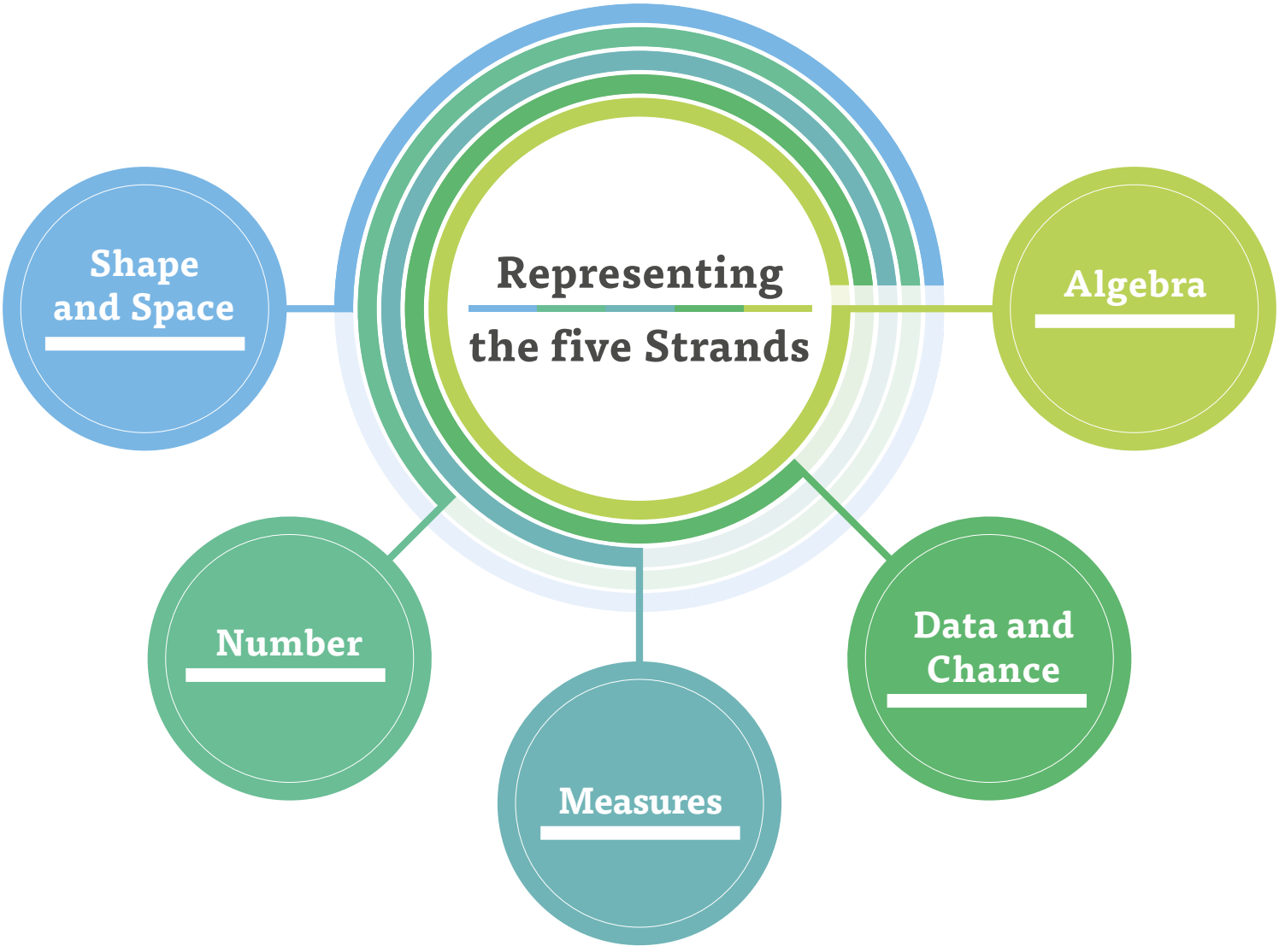


Figure 7: Representing the five Strands

Strands	Learning Outcome Labels
Algebra	Patterns, Rules and Relationships Expressions and Equations
Data and Chance	Data Chance
Measures	Measuring Time Money
Number	Uses of Number Numeration and Counting Place Value and Base Ten Sets and Operations Fractions
Shape and Space	Spatial Awareness and Location Shape Transformation

Table 1: Overview of the Learning Outcome Labels

Curriculum Elements

Children develop their mathematical proficiency through engaging with processes such as connecting, communicating, reasoning, argumentation, justifying, representing, problem solving, and generalising. These mathematical processes are categorised into four Elements: Understanding and Connecting; Communicating; Reasoning; Applying and Problem solving.

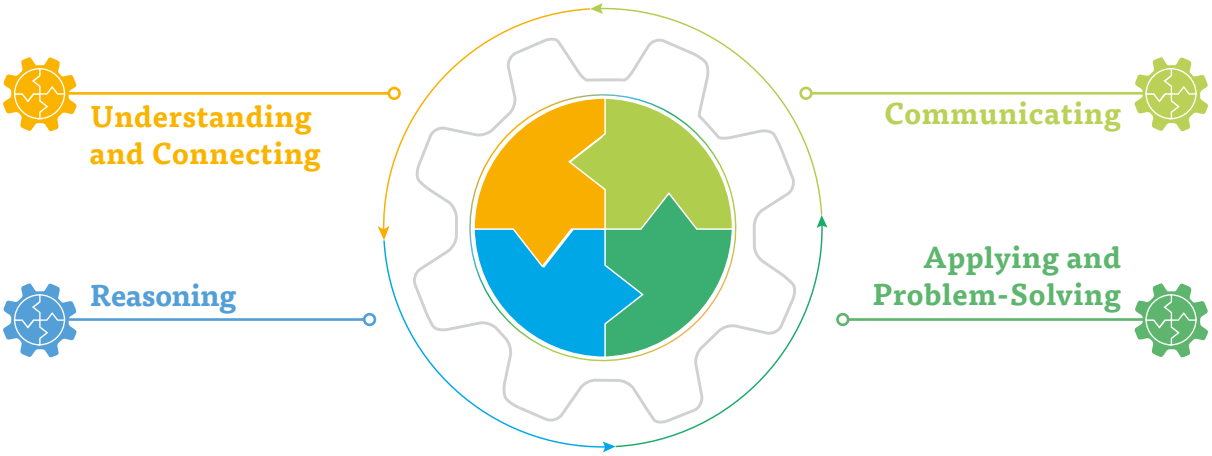


Figure 8: The elements of the Primary Mathematics Curriculum

Element 1: Understanding and Connecting

Children should make connections between related concepts and procedures—the 'why' and the 'how' of mathematics—and between new and prior knowledge, in order to make sense of what they are learning. They should apply and connect their understanding to contexts within mathematics, with other areas of learning, and with the real world.

Element 2: Communicating

Children should use appropriate means of communication and a variety of representations and conventions to convey thinking, ideas, relationships and logical arguments. They should improve and refine their thinking and communication through engaging in inquiry-based learning and social learning environments.

Element 3: Reasoning

Children should develop and apply reasoning to make, assess and justify ideas and conjectures. They should engage in logical thought and actions such as analysing, proving, inferring and generalising. They should plan and construct solid arguments to justify their explanations, proofs and decision making.

Element 4: Applying and Problem Solving

Children should investigate, develop, select, apply, interpret, model and compare a variety of problem-solving situations and strategies as they explore mathematics and deepen their mathematical understanding. They should apply their mathematical knowledge and skills in flexible, efficient and creative ways to solve problems, conduct investigations and develop their computational thinking.

5. Learning Outcomes

Learning Outcomes are used to describe the expected mathematical learning and development for all learners at the end of a two-year stage, when due account is taken of individual abilities and varying circumstances. They articulate big mathematical ideas across each stage; and encompass the knowledge, skills and dispositions that children should aim to demonstrate. Reflecting *Aistear's* principles and pedagogical approaches, the stem, 'Through appropriately playful learning experiences, children should be able to…' is used to introduce Stage 1 Learning Outcomes, clearly indicating a playful approach to teaching and learning in the early years of primary school. For all other stages, the stem 'Through appropriately engaging learning experiences, children should be able to…' is used. This stem indicates the importance of providing rich learning experiences that reflect relevant pedagogical approaches as outlined in Chapter 6 'The Primary Mathematics Curriculum in Practice'.

The curriculum recognises that children learn, and teachers teach, in a variety of contexts and so teaching/learning journeys will be different in these contexts. Learning Outcomes, when shared with children, can support them to hold clear expectations and to be active agents in their own learning. Moreover, Learning Outcomes allow for teacher agency and flexibility in exercising professional judgement and decision-making around planning, teaching and assessment for individual contexts. A Learning Outcomes approach recognises that teachers are best placed to determine the learning needs of the children in their class, what and how to teach using appropriate pedagogical approaches as well as what and how to assess based on the intended learning.

Learning Outcomes help teachers to

- prepare for, teach and reflect on their use of appropriate methods for teaching and learning mathematical ideas
- focus the use of assessment to gather evidence of children's learning and understanding, thereby enabling teachers to adapt their teaching and respond appropriately to children's learning
- provide focused feedback to children and parents/ guardians.

Learning Outcomes for Algebra Strand

	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)	Stage 3 (3rd & 4th Class)	Stage 4 (5th & 6th Class)
Patterns, Rules and Relationships	*Through appropriately playful learning experiences children should be able* to explore, extend and create patterns and sequences.	*Through appropriately engaging learning experiences children should be able* to identify and express relationships in patterns, including numerical and shape patterns.	*Through appropriately engaging learning experiences children should be able* to identify the rules that govern patterns to generalise and make predictions. *Through appropriately engaging learning experiences children should be able* to represent the relationships between quantities.	*Through appropriately engaging learning experiences children should be able* to identify, explain and apply generalisations, including properties of operations, mathematical models and patterns. *Through appropriately engaging learning experiences children should be able* to represent mathematical structures in multiple ways, including verbal expressions, diagrams, and symbolic representations.
Expressions and Equations		Through appropriately engaging learning experiences children should be able to interpret the representative nature of symbols or pictures in number sentences or expressions.	*Through appropriately engaging learning experiences children should be able* to represent and express problems with unknown values in different ways to include the use of appropriate letter-symbols or words.	*Through appropriately engaging learning experiences children should be able* to articulate, represent and solve mathematical situations through the use of expressions and equations that include letter-symbols.

Mathematical processes, as described in the curriculum elements (See page 17), are critical for children's learning experiences.

Learning Outcomes for Data and Chance Strand

	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)	Stage 3 (3rd & 4th Class)	Stage 4 (5th & 6th Class)
Data	*Through appropriately playful learning experiences children should be able* to explore, interpret and explain data in a variety of ways for a range of purposes.	*Through appropriately engaging learning experiences children should be able* to pose questions of interest, record and use data as evidence to answer those questions, and communicate the findings.	*Through appropriately engaging learning experiences children should be able* to pose questions of interest and collect, display and critically analyse data in a range of ways for a range of purposes, and communicate the findings.	*Through appropriately engaging learning experiences children should be able* to pose questions, collect, use and record data selectively to answer those questions. *Through appropriately engaging learning experiences children should be able* to critically analyse and evaluate findings and; communicate inferences, conclusions and implications from the findings.
Chance			*Through appropriately engaging learning experiences children should be able* to describe and test predictability and (un) certainty in events.	*Through appropriately engaging learning experiences children should be able* to use probability to make informed decisions and predictions. *Through appropriately engaging learning experiences children should be able* to represent and express probability in different forms.

Mathematical processes, as described in the curriculum elements (See page 17), are critical for children's learning experiences.

Learning Outcomes for Measures strand

	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)	Stage 3 (3rd & 4th Class)	Stage 4 (5th & 6th Class)
Measuring	*Through appropriately playful learning experiences children should be able* to demonstrate an awareness of measuring length, weight (mass), capacity and area and its purpose in comparing the attributes of objects.	*Through appropriately engaging learning experiences children should be able* to compare, approximate and measure length, weight, capacity and area using appropriate instruments and record using appropriate units of measurement.	*Through appropriately engaging learning experiences children should be able* to compare, estimate and measure length, weight, capacity, area and volume using appropriate instruments and record and communicate appropriately. *Through appropriately engaging learning experiences children should be able* to identify the relationship between equivalent units of measurement, and rename measures using equivalent units.	*Through appropriately engaging learning experiences children should be able* to determine and calculate units of measurement in fractional and/ or decimal form to solve practical problems. *Through appropriately engaging learning experiences children should be able* to find, interpret and deduce measures experimentally with increasing precision.
Time	*Through appropriately playful learning experiences children should be able* to develop a sense of time and its purpose.	*Through appropriately engaging learning experiences children should be able* to understand how time is measured, expressed and represented. Through appropriately engaging learning experiences children should be able to explore equivalent expressions of time.	*Through appropriately engaging learning experiences children should be able* to compare, approximate and measure time using appropriate units of measurement. Through appropriately engaging learning experiences children should be able to identify the relationship between different units and representations of time.	*Through appropriately engaging learning experiences children should be able* to solve and pose practical tasks and problems involving the interpretation and calculation of time.
Money	*Through appropriately playful learning experiences children should be able* to develop awareness of money and its purpose.	*Through appropriately engaging learning experiences children should be able* to recognise the value of money and use euro and cent in a range of meaningful contexts.	*Through appropriately engaging learning experiences children should be able* to transfer knowledge of the base ten system in number to monetary contexts, and use for purposes of calculation.	*Through appropriately engaging learning experiences children should be able* to solve and pose practical tasks to investigate and make informed judgements about transactions and financial plans.

Mathematical processes, as described in the curriculum elements (See page 17), are critical for children's learning experiences.

Learning Outcomes for Number strand

	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Uses of Number	*Through appropriately playful learning experiences children should be able* to develop an awareness that numbers have a variety of uses.			
Numeration and Counting	*Through appropriately playful learning experiences children should be able* to develop an awareness that the purpose of counting is to quantify. *Through appropriately playful learning experiences children should be able* to use a range of counting strategies for a range of purposes.	*Through appropriately engaging learning experiences children should be able* to demonstrate proficiency in using and applying different counting strategies.		
Place Value and Base Ten	*Through appropriately playful learning experiences children should be able* to develop a sense of ten as the foundation for place value and counting.	*Through appropriately engaging learning experiences children should be able* to understand that digits have different values depending on their place or position in a number. Through appropriately engaging learning experiences children should be able to use estimation to quickly determine the value and calculation of numbers.	*Through appropriately engaging learning experiences children should be able* to explore equivalent numerical expressions of numbers using the base ten system.	*Through appropriately engaging learning experiences children should be able* to investigate how fractions, decimals and percentages can be compared, ordered and expressed in related terms.

Learning Outcomes for Number strand (Continued)

	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)	Stage 3 (3rd & 4th Class)	Stage 4 (5th & 6th Class)
Sets and Operations	*Through appropriately playful learning experiences children should be able* to recognise and understand what happens when quantities (sets) are partitioned and combined.	*Through appropriately engaging learning experiences children should be able* to select, make use of and represent a range of addition and subtraction strategies.	*Through appropriately engaging learning experiences children should be able* to understand and apply flexibly the four operations; and the relationships between operations.	*Through appropriately engaging learning experiences children should be able* to build upon, select and make use of a range of operation strategies.
Fractions	*Through appropriately playful learning experiences children should be able* to develop an awareness of part-whole relationships using a variety of models (area, length and set).	*Through appropriately engaging learning experiences children should be able* to recognise and name fractions according to their part-whole relationships. *Through appropriately engaging learning experiences children should be able* to explore the concept of equivalence in terms of simple fractions.	*Through appropriately engaging learning experiences children should be able* to compare, express in equivalent terms, and order fractions. *Through appropriately engaging learning experiences children should be able* to calculate the fraction of quantities and express in multiple ways.	*Through appropriately engaging learning experiences children should be able* to explore (model, compare and convert) the relationships between fractions, decimals and percentages. *Through appropriately engaging learning experiences children should be able* to investigate proportionality and ratios of quantities (sets).

Mathematical processes, as described in the curriculum elements (See page 17), are critical for children's learning experiences.

Learning Outcomes for Shape and Space strand

	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)	Stage 3 (3rd & 4th Class)	Stage 4 (5th & 6th Class)
Spatial Awareness and Location	*Through appropriately playful learning experiences children should be able* to develop a sense of spatial awareness in relation to their bodies and immediate environment. *Through appropriately playful learning experiences children should be able* to describe the spatial features of objects and people, and their relative position in space.	*Through appropriately engaging learning experiences children should be able* to use spatial knowledge for the purposes of orientation, way-finding and navigation. *Through appropriately engaging learning experiences children should be able* to visualise and model location using symbolic co-ordinates.	*Through appropriately engaging learning experiences children should be able* to describe and interpret directional instructions. *Through appropriately engaging learning experiences children should be able* to compare and classify angles, recognising them as a property of a shape and as a description of a turn.	*Through appropriately engaging learning experiences children should be able* to describe location on the full co-ordinate plane. *Through appropriately engaging learning experiences children should be able* to investigate and construct angles in different contexts, and solve angle-related problems.
Shape	*Through appropriately playful learning experiences children should be able* to explore and recognise properties of 3-D and 2-D shapes.	*Through appropriately engaging learning experiences children should be able* to examine, categorise and model 3-D and 2-D shapes.	*Through appropriately engaging learning experiences children should be able* to investigate and analyse the properties and calculate dimensions of shapes.	*Through appropriately engaging learning experiences children should be able* to construct models or structures given defined measurements and/ or specific conditions.
Transformation	*Through appropriately playful learning experiences children should be able* to explore the effects of shape movements.	*Through appropriately engaging learning experiences children should be able* to understand that shapes and lines can be reflected, rotated, dilated and translated.	*Through appropriately engaging learning experiences children should be able* to model and explain the effects of transformations on shapes and lines.	*Through appropriately engaging learning experiences children should be able* to perform and devise a range of steps involving transformations.

Mathematical processes, as described in the curriculum elements (See page 17), are critical for children's learning experiences.

6. The Primary Mathematics Curriculum in Practice

As outlined in chapters 2 and 3, the rationale and aims of the Primary Mathematics Curriculum offer a new vision for children's learning in mathematics. To assist teachers in bringing the curriculum to life, Examples of Children's Learning and Support Materials for Teachers are provided in the Primary Mathematics Toolkit (see Chapter 7 for more information). In addition to these, this chapter describes five key pedagogical practices which underpin and embody this new vision for children's learning in terms of teachers' everyday practice.

Strongly rooted in research,[2] these five key pedagogical practices (see Figure 8) promote the development of mathematical proficiency and, as such, are central to the successful enactment of the curriculum in the classroom.

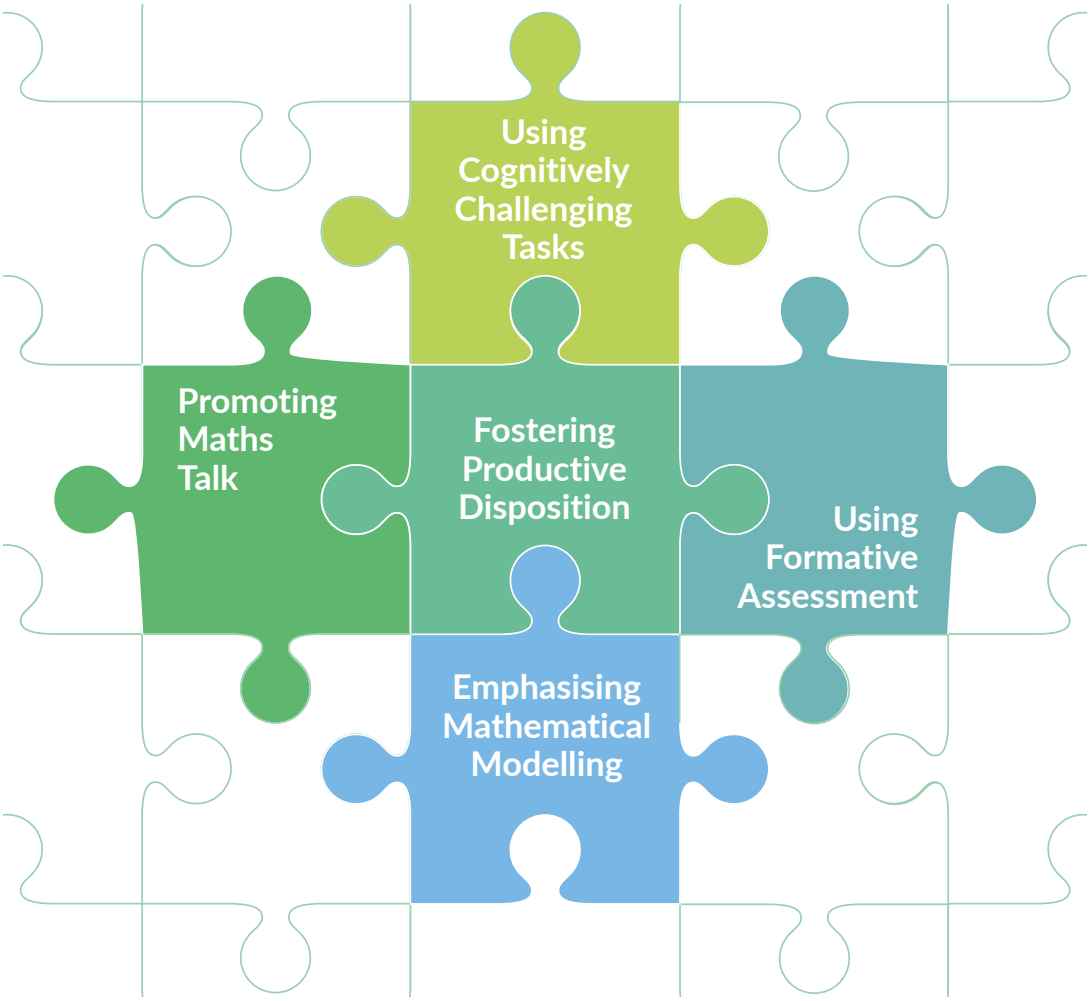


Figure 9: Five key pedagogical practices for the classroom

[2] *N.B. For more information on the research which underpins these five pedagogical practices, please see www.ncca.ie

'How' children learn is as important as 'what' children learn in mathematics. The five pedagogical practices presented here are essential to the provision of quality learning experiences in the classroom. They can foster an inclusive learning environment and culture where children can engage in rich and meaningful learning processes such as understanding and connecting; communicating; reasoning; and applying and problem-solving. Suggested learning experiences for children which incorporate these core processes can be found in the Progression Continua (see Primary Mathematics Toolkit). Moreover, these pedagogical practices allow for children to learn and develop at a pace and level of challenge that is individual to their needs and interests whilst developing their confidence, interest and proficiency in mathematics. As such, these practices should permeate teachers' everyday preparation for teaching and learning of mathematics as well as their approaches to assessment.

Interconnectivity

As with most good classroom practices, these practices are dynamic and naturally link with each other. For example, when teachers place an emphasis on Mathematical Modeling, children can be enabled to express and make visible their conceptual understanding. They are presented with the opportunity to use formative assessment to gain deeper insights into children's level of understanding. Similarly, by using Cognitively Challenging Tasks, teachers can open up rich opportunities for children to engage in Maths Talk as they discuss, refine and justify their ideas and solution paths.

This chapter provides an outline of each of these five pedagogical practices and how each might benefit children's learning; as well as suggestions for how these practices might be incorporated into classroom practice. It is important to note that this list of pedagogical practices is neither exhaustive nor hierarchical.

Fostering Productive Disposition

Dispositions are not static and can be nurtured or changed over time. The multiple ways in which children engage with mathematics, how they perceive mathematics, and the rich contexts in which mathematics is meaningfully presented to them are what helps form and shape their disposition for this subject. In addition, attitudes to mathematics and values, both at home and in the classroom, also have a strong impact on the development of the child's productive disposition for mathematics. A classroom which emphasises the rich, useful and meaningful nature of mathematics has a very positive effect on the child's disposition to learning.

When children have a productive disposition to mathematics, they

- see it as worthwhile, useful and 'doable'
- tend to demonstrate enthusiasm and curiosity for mathematics
- are more likely to be diligent and resilient in their work
- persevere in challenging mathematical tasks and problems
- see mistakes as opportunities to learn something new or deepen their understanding
- take responsibility for their own learning.

Fostering a productive disposition to mathematics is a shared responsibility

Families, teachers and other significant adults in the child's life all play a role in presenting a positive view of mathematics. When children see the application of mathematics in situations they care about and which are relevant to their lives, it is more likely that they will engage with mathematics in a meaningful way. Teachers can promote the development of productive disposition in the home by suggesting fun and meaningful mathematical activities and games that can be used with children.

Many everyday activities that children enjoy offer opportunities for a rich engagement with mathematics. There are a number of supports available online for families to support positive and purposeful engagement with mathematics in the home environment.

Suggestions for developing productive dispositions to mathematics

The practices used in classrooms should enable children's agency; take into account their individual needs and strengths; incorporate their interests and learning preferences; and allow children to experience success in their learning regardless of their mathematical ability. Teachers can promote the development of productive disposition in a number of ways, such as by

- demonstrating enthusiasm for mathematics themselves
- celebrating children's individual and collaborative successes
- encouraging active participation and sustained effort
- providing scaffolding, encouragement and support
- encouraging children to take risks and persevere in overcoming challenges
- facilitating children to work collaboratively in mixed-ability groups, setting clear expectations and roles for group members
- praising effort and creative strategies
- valuing struggle and normalising mistakes as steps towards deeper understanding
- valuing the process as well as the product
- promoting exploration, investigation and questioning
- encouraging children to notice, explore and discuss patterns and connections
- soliciting multiple solution strategies and approaches
- allowing children time to think and reflect
- utilising low-threshold high-ceiling tasks
- engaging in meaningful self-assessment and reflection throughout the learning process.

Emphasising Mathematical Modeling

Mathematical modeling involves using mathematics to describe a problem-context and determine meaningful solutions to the problem. Children form models through a process of testing, revising and expressing their interpretation of different mathematical ideas, experiences, problems and situations; typically posed to them as questions or challenges. Children naturally generate their own informal mathematical models in a way that is context-specific and makes sense to them. As children's knowledge, understanding and experience grows, they may develop more formal, sophisticated and efficient models which they can use to share, connect and communicate their ideas with others, and which they may transfer to a range of different contexts in a way that is meaningful to themselves and others. In forming models, children might use physical actions, spoken words, objects, images (e.g., graphs, diagrams and pictures), symbols or written words. While accuracy is valued, Mathematical Modeling places more importance on exploration, sense-making, conceptual understanding and flexibility in thinking.

By engaging in Mathematical Modeling, children can

- formalise their learning of mathematics in a way that makes sense to them
- express and make explicit their ideas and experiences
- deepen their understanding of complex mathematical concepts, problems and situations
- take ownership of their learning
- interpret and analyse problems
- explore, conjecture, compare, and justify ideas and solutions
- test and refine various models and make predictions based on evidence
- transfer and apply formal mathematical knowledge to new problems and situations.

Suggestions for emphasising Mathematical Modeling

When appropriately facilitated in the curriculum, Mathematical Modeling can enable children to solve problems or dilemmas in meaningful and efficient ways. It also provides opportunities for teachers to collaborate with children at a deeper level and to assess how children apply their conceptual knowledge in mathematical situations. Teachers can emphasise Mathematical Modeling in a number of ways, such as by

- providing time and space for children to make sense of their thinking, and revisit and refine their ideas and solutions
- refraining from imposing personal models and particular paths to a solution, unless appropriate
- using questions, prompts and feedback to provoke situations that encourage children to model mathematically
- engaging children in model-eliciting activities which allow for multiple interpretations and representations
- encouraging individual approaches, choice and independence
- providing a wide range of examples (for example, use of diagrams or graphs) and resources (for example, blocks, empty number lines or bar models) that might help children to scaffold, develop and express their own models
- facilitating children to use technology in order to build, test and apply mathematical models
- challenging children to express their understandings, to test their thinking and to revise their models as often as necessary
- over time, supporting children to generalise their personal models to a range of different contexts for a range of different purposes.

Using Cognitively Challenging Tasks

Cognitively Challenging Tasks are rich higher-order learning opportunities that should appropriately stretch and challenge children's conceptual understanding as they encounter significant mathematical ideas and situations. Sometimes referred to as low-threshold high-ceiling tasks, these tasks should provide all children with the opportunity to access mathematics, while offering the potential for deeper engagement. Simple, considerate and well-pitched tasks can present a rich medium through which children can engage meaningfully in mathematical content and processes; and which also offer opportunities for teachers to incorporate other key pedagogical practices.

When used effectively, children perceive and experience these tasks as having few prescribed or memorised rules or methods, and an opportunity to explore multiple solution paths. It is through exploring these tasks and grappling with problems and solutions that children deepen their understanding of mathematics, and the sophistication with which they can engage with mathematical ideas and situations. Through their efforts to engage with Cognitively Challenging Tasks, children develop persistence and resilience, essential to their productive disposition for mathematics.

Through engaging with and experiencing Cognitively Challenging Tasks, children

- see their struggles as valuable opportunities to deepen their understanding of mathematics, experience productive struggle, grapple with problems and explore different strategies and solutions
- use a variety of mathematical models to demonstrate and communicate their ideas and solutions
- discover patterns and structures that help to make their learning more meaningful
- engage with mathematics in meaningful and relevant contexts
- identify personal methods and better understanding of themselves as learners
- experience individual success and satisfaction
- give and receive feedback that can enrich and deepen their learning
- become more self-reliant and self-assured, not depending solely on the teacher for confirmation of their proficiency in mathematics.

Suggestions for the provision of Cognitively Challenging Tasks

In a classroom that facilitates deep engagement with Cognitively Challenging Tasks, the learning space should offer flexible opportunities for children to explore different strategies, models and responses. Mistakes, erroneous solutions and diverse approaches should be expected, welcomed and respected. Learning paths that are open and flexible involve a degree of unpredictability. Equally, when children are grappling with these tasks, there may be a sense of quietness and uncertainty for teachers to contend with. A culture of Maths Talk and an emphasis on Mathematical Modeling and productive disposition, lend strongly to positive engagement with Cognitively Challenging Tasks. Tasks can be used to introduce new topics; to reinforce and/or build on prior learning; or in response to contingency moments.

Teachers can facilitate Cognitively Challenging Tasks in the classroom by

- selecting, designing or modifying tasks that are appropriate for the intended learning
- presenting tasks as a medium for deep and sustained engagement with mathematical content and processes
- refraining from proceduralising the task, for example, 'first you should...' and rather allow children to explore solution paths and ideas freely
- providing appropriate time and space, support materials and/or scaffolded support to allow children to grapple with ideas and problems
- assisting children to make connections between prior and new knowledge; and different areas of mathematics
- encouraging children to express and communicate their ideas frequently and openly
- avoiding emphasising one particular correct solution or path, intervening too early, rescuing the learner, or reducing the cognitive demand of the task
- providing opportunities for children to collectively discuss, compare, justify and evaluate their ideas, pathways or solutions
- celebrating individual and collaborative effort and success in grappling with challenging tasks.

Promoting Maths Talk

Maths Talk is a collaborative process where children's thinking, strategies and ideas are discussed, shared and / or exchanged. This allows children to reflect on their own understanding; define, present and justify their ideas; make sense of and critique their own ideas and those of others; and develop their ability to express and articulate their thinking. Through Maths Talk, children can engage in rich mathematical processes which deepen their understanding of mathematics. For example, by presenting, arguing or justifying their mathematical ideas they can refine, consolidate or extend their existing knowledge.

Maths Talk helps children in a number of ways, such as to

- elicit thinking and link prior knowledge to new learning
- articulate the reasoning behind their thinking
- clarify their thinking and rehearse their ideas
- discuss new possibilities and make conjectures
- refine and deepen their understanding
- explore and justify their ideas and solution
- extend their own thinking and the thinking of others
- develop confidence in articulating their ideas and misconceptions.

Maths Talk equips children with tools to make their thinking visible. These tools include words, body language and gestures, signs, symbols, diagrams, concrete manipulatives and technology (see Figure 13). For children who may have difficulties with communication, opportunities should be provided for children to communicate in a way that is meaningful for them, for example using gestures, Lámh or perhaps pictures. All children are mathematics language learners, regardless of their language proficiency, and can engage in Maths Talk. The learning environment should be flexible in terms of recognising and catering for multiple forms of expression and multiple means of engagement. The physical space should provide children with access to a variety of materials that stimulate and enable Maths Talk.

In turn, Maths Talk has a number of pedagogical benefits by helping teachers to

- determine children's dispositions and gauge their developing confidence, interest, and proficiency in mathematics
- identify misconceptions held by the children
- co-construct and refine ideas in collaboration with children
- make explicit links between the ideas of different children and promote collaboration
- identify strategies and approaches that best suit the individual child
- respond to contingency moments
- gather assessment information that can be used to inform planning and teaching
- monitor children's development and progress.

Suggestions for promoting Maths Talk

Opportunities for Maths Talk can be planned but are also likely to occur organically. Teachers can promote and encourage Maths Talk in a number of ways, such as by

- fostering a learning culture where children feel safe to share and exchange their thinking and ideas
- creating a space where children are encouraged to actively listen to, respect and value the input of others
- planning for distinct activities to elicit discussion and/ or develop and assess explicit Maths Talk skills
- expressing interest and curiosity in children's interests, descriptions, explanations, justifications and solutions
- identifying and selecting appropriate situations and problems to generate Maths Talk
- re-casting everyday experiences using mathematical words and phrases
- introducing and incorporating appropriate new language to allow for meaningful engagement with topics
- giving children opportunities to interact and work collaboratively with their peers
- strategic, skilful, open and thoughtful questioning
- encouraging children to take risks and try out new strategies and approaches
- acknowledging mathematically correct responses whilst also welcoming, encouraging and normalising multiple understandings, mistakes and misconceptions
- providing suggestions for parents on how to promote and stimulate Maths Talk at home allowing waiting time and time for sustained interactions
- promoting collective sharing and reflection
- re-voicing children's ideas, where appropriate.

Using Formative Assessment

Assessments are formative when evidence about children's progress is observed, interpreted, and used by teachers or children in a responsive way, to make decisions on an ongoing basis about the next steps in their mathematical learning journey. The more that is understood about where children are at in their learning and how they learn, the better their mathematical understanding and productive disposition can be nurtured, and the learning environment tailored to meet the needs of all learners.

Formative assessment is consistent with a child-centred approach to mathematics and when used effectively can help children to become more independent and motivated. Children are experts in their own learning. By equipping children with appropriate skills, and by sharing learning intentions and agreeing criteria for successful learning with children, teachers can give children the tools they need to peer- and self-assess, reflect on and take greater responsibility for their own learning. Feedback may come from informal (e.g. observations) and formal (e.g. standardised tests) sources.

The ability to recognise the mathematics in children's everyday activities and to extend the potential learning arising from these is critical to planning for formative assessment in the classroom. However, while formative assessment can be planned, it is also likely to occur incidentally, intuitively and in response to contingency moments (unplanned and unexpected responses from a learner) during the learning process. Providing an openness for such contingency moments, and capitalising where appropriate on insights gleaned from children's responses, questions, problems and tasks, can allow for very rich assessment data. The Progression Continua, found in the Primary Mathematics Toolkit, may be a useful tool for teachers in planning for formative assessment in the classroom.

Suggested methods for incorporating Formative Assessment

As teachers continually assess children's mathematical knowledge, skills and dispositions, teachers are likely to use multiple formative assessment methods, in inter-changeable ways, to build a rich picture of children's mathematical learning. Some of these formative assessment methods include

Observations: Teachers are well placed to actively monitor and observe children as they engage in learning experiences and incorporate insights from these observations in future planning and/ or in response to opportune learning situations as they arise organically.

Feedback: Through interactions and the provision of feedback, focused on the learning or task in hand, this can help children identify and celebrate their progress and achievements, pinpoint challenges they experience, give direction for future work and decide what the appropriate next steps might be.

Tasks: Tasks can be very useful to provide an insight into children's level of understanding and personal methods of learning. Assessment tasks can be written, oral or practical.

Conferencing: Through teacher/ child, peer/ peer and group conferencing, teachers can gain an insight into children's mathematical thinking and the nuances of their learning and their experience of learning.

Portfolios: Portfolios can be assembled, digitally or otherwise, to compile evidence of children's mathematical learning and provide a source of self-reflection, feedback and assessment. Artefacts could include pictures, recordings and work samples etc.

Questioning: Effective, thoughtful and well-crafted questions that are open, related to the lesson learning intentions, and accompanied by appropriate wait time; encourages children to think deeply, develop their understanding and express their ideas and solutions.

Formative use of summative tests: Summative tests can provide an important source of detailed feedback when used to inform and guide future planning and teaching.

Suggested methods for incorporating Formative Assessment

Peer and Self-Assessment Methods

Teachers may also use strategies to support children to engage in peer and self-assessment and rely increasingly less on extrinsic affirmation and motivation. An open, respectful and collaborative classroom culture and learning environment are essential to supporting children to think about their own learning and that of others. Children working in pairs or small groups may be necessary to facilitate group reflection and feedback sharing. Displaying children's work to allow for peer and self-assessment over time may also be useful.

Peer and self-assessment can also be promoted by

- allowing time and space for children to reflect and discuss their learning
- making explicit to children what they are going to learn, how they might build on prior knowledge and possible connections with other areas of mathematics familiar to them
- suggesting criteria for children to use in monitoring their progress
- promoting and celebrating respectful and open collaboration, reflection and sharing.

7. Primary Mathematics Toolkit

The Primary Mathematics Toolkit provides practical support for teachers in building rich mathematical learning experiences for children. The four components of the toolkit are described in the following section – Mathematical Concepts, Progression Continua, Support Materials and Examples of Children's Learning.

7.1 Mathematical Concepts

Mathematical Concepts are considered essential ideas that underpin each Learning Outcome. These essential ideas may provide useful entry and reference points in relation to planning, teaching and assessment and may serve to remind teachers of key mathematical knowledge at each stage. The Mathematical Concepts are situated in the Primary Mathematics Toolkit and can also be found in the appendix of the main specification. They are presented according to stages 1 to 4 and link with the corresponding Learning Outcomes. Children should develop their understanding of these corresponding mathematical concepts through engaging with a wide variety of mathematical processes.

7.2 Progression Continua

The Progression Continua providea focus for supporting children's learning and development, by suggesting how children might engage with mathematical content through increasingly sophisticated skills. Classrooms are complex and children come to class with different experiences and learn in diverse ways. They also learn and develop at different rates. Meeting the learning needs of all children within a classroom means the provision of differentiated learning experiences. The Progression Continua provideguidance for the provision of such differentiation by suggesting a wide range of learning experiences with which children might engage.

While the Progression Continua suggest a typical learning journey in mathematics throughout primary school, they are not intended to be prescriptive or exhaustive. As such, teachers should exercise professional judgement when making decisions as to the learning experiences which are most appropriate for the children in their classroom. This recognises that not all children learn in a linear or typical way. Children may move forwards and backwards across the continua for different strands or elements of mathematical learning, while others may work within one or across a small number of progression steps for the duration of their primary school years.

While the Progression Continua provide a useful reference for teachers, decisions on the kinds of learning experiences that are most appropriate for one's class should be primarily evidence-based. Evidence may indicate the need to reinforce, revisit or extend a particular concept or element, and should support planning as well as holistic assessment of children's learning. In gathering evidence to determine a starting point for children's learning experiences at the beginning of the school year, it may be useful to select one of the learning experiences suggested in the Progression Continua, and for example, observe and ask probing questions relating to the Learning Outcome.

There are fifteen Progression Continua tables, one for each of the Learning Outcome Labels. Each continuum describes the learning journey across eleven Progression Milestones (a-k). Progression Milestones describe learning in terms of mathematical content and processes. In Progression Continua, mathematical processes are categorised as four key Elements – Understanding and Connecting; Communicating; Reasoning; Applying and Problem-Solving.

7.3 Support Materials

The Primary Mathematics Toolkit will include a range of Support Material for teachers to use with the Primary Mathematics Curriculum. The Support Material for teachers in the Toolkit will include important resources and reference material including research reports and articles, content related information, pedagogical supports and advice, news and updates.

It is hoped that teachers will refer to the variety of Support Material provided and consequently reflect on how they might implement and adapt the practices shown to support their own interpretation, professional learning and enactment of the curriculum in practice. The Toolkit will be a rich resource for supporting practice and it is anticipated that it will continue to be populated over a period of time as needs, new resources and reference materials for teaching mathematics are identified.

7.4 Examples of Children's Mathematical Learning

The Primary Mathematics Toolkit will include a range of Examples of children's mathematical learning and development. These Examples will illustrate teacher-child interactions, Cognitively Challenging Tasks, problem-based learning, learning in integrated contexts, amongst others. In doing so, it is intended that these examples will exemplify rich and engaging learning experiences and demonstrate good pedagogical practices in supporting children towards achieving learning outcomes and developing their mathematical proficiency.

8. Glossary

The glossary is currently under development. It highlights key words and terminology used in the curriculum which may be new to teachers or which may require further explanation. The upcoming consultation with teachers and stakeholders will provide a good opportunity to finalise the list of words which should be included in this glossary.

Argumentation: A dynamic process for discovering and understanding new mathematical ideas and presenting the rationale for same.

Child agency: Children are active in their own learning. Some ways in which they display their agency is by taking the initiative in learning situations, by observing and becoming involved in ongoing events, or by initiating conversations with others.

Collaborative learning: Learning that takes place in social contexts and using the resources of the environment.

Conceptual understanding: Understanding of mathematical concepts, operations and relations.

Conferencing: Dedicated time, space or meeting to elicit and gain a deeper insight into children's level of understanding.

Conjecture: An educated guess that is based on known and/ or incomplete information.

Contingency moments: Unplanned or unexpected events or responses from children that occur during the learning process.

Co-ordinate plane: A two-dimensional plane divided into four quadrants.

Dilation: A shape or line is dilated when it's size changes (gets larger or smaller), but its position remains the same.

Disposition: An enduring habit of mind and action. The tendency to respond to situations in characteristic ways.

Formative assessment: Assessment is formative when either formal or informal procedures are used to gather evidence of learning, and subsequently respond to this during the learning process.

Generalising: To make assertions, claims or justifications as to how their understanding is applicable or transferrable to other circumstances.

Learning environment: Describes any space in which children learn or develop their understanding.

Learning intentions: Identify new learning that will take place in a lesson or activity.

Low-threshold high-ceiling tasks: Provide accessible entry points for learning with the scope for exploration and challenge for all learners.

Mathematical journals: A mechanism for children to record their mathematical work and thinking.

Mathematical Modeling: Mathematical modeling involves using mathematics to conceptualise a problem or situation and determine meaningful solutions, and in doing so help children formalise their mathematical learning in a way that makes sense to them.

Mathematical proficiency: Consists of the five intertwined and interrelated strands of conceptual understanding, procedural fluency, strategic confidence, adaptive reasoning and productive disposition.

Maths Talk: A collaborative process where children's thinking, strategies and ideas are discussed, shared and exchanged.

Peer assessment: Involves children looking at each other's work in a reflective way.

Productive disposition: The inclination to see mathematics as something worthwhile, useful and doable.

Productive struggle: This is the process of effortful learning whereby children develop grit, perseverance and flexible thinking as they engage with new and demanding ideas and situations.

Re-voicing: The teacher repeats some or all of what the child has said and then asks the child to clarify whether or not this may be correct.

Rotation: Turning around a centre point.

Scaffolding: Describes the process by which teachers support and guide children's learning, by building on **their current knowledge and experience.**

Self-assessment: Involves children looking at their own work in a reflective way.

Summative assessment: Assessment is summative when it is used to evaluate children's learning at the end of the instructional process or of a period of learning.

Symbolic co-ordinates: Used to describe the position or location of a point or object.

Translation: A shape or line is translated when it is moved a certain distance from its original position (without turning).

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10. Appendices

Appendix 1: Mathematical Concepts

Mathematical Concepts are considered essential ideas that underpin each Learning Outcome. These essential ideas may provide useful entry and reference points in relation to planning, teaching and assessment, and may serve to remind teachers of key mathematical knowledge at each stage.

The Mathematical Concepts are situated in the Primary Mathematics Toolkit and can also be found in the appendix of the main specification. They are presented according to stages 1 to 4 and link with the corresponding Learning Outcomes. Children should develop their understanding of these corresponding mathematical concepts through engaging with a wide variety of mathematical processes.

The Progression Continua provides a comprehensive outline of suggested learning experiences to support children in their learning and development through a wide variety of mathematical processes.

Patterns, Rules and Relationships				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	A pattern is an arrangement of elements organised according to a defined structure or rule.	Quantitative change in growing and shrinking patterns is consistent and predictable.	A purpose of representations is to show relationships between quantities.	A mathematical expression expresses the structure of a pattern.
	A sequence is a list of objects or actions that follow an order.	When we identify the structure or rules governing a pattern, we can use this to identify, express, predict and generalise change or variation.	Representing patterns in words, number sentences, diagrams and graphs can be useful to solve problems or to determine values for unknown or future events.	Describing a real-life situation using words or symbols can be useful to solve problems or to determine values for unknown or future events.
	There are patterns and sequences to everyday routines and events, from which predictions can be made.	From exploring the structure of patterns, one can determine what is changing and what is staying the same.	A geometric sequence of numbers is based upon multiplication or division. Each consecutive number is found by multiplying the preceding number by a common multiplier.	An integer is a whole number that can be positive, negative, or zero.
	Repeating patterns are made up of units of repeat and do not contain random elements.	Number patterns tell the story of relationships between quantities. The relationship between elements of a pattern can be expressed using word or number sentences.	The associative property is a rule that says the way in which numbers are grouped in an addition or multiplication sentence does not change the sum or product.	Whole numbers can be expressed in standard form, factor form and be written as a product of its factors.
	The zero property of addition and subtraction means that when you add zero to or subtract zero from a number it does not change the number.	Patterns can be used to determine number facts we don't know from number facts we do know.	The distributive property can be used to simplify complex multiplication and division equations by breaking one (in the case of the dividend) or both numbers down into smaller parts.	A square number is what we get after multiplying an integer by itself. A square root can be multiplied by itself to get the original number.
		The commutative property of addition means we can swap the order of the numbers being added and still get the same total.		

Expressions and Equations				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts		Real-life situations can be expressed using manipulatives, diagrams, and word and number sentences.	When expressing real-life situations, symbols can be used to represent an unknown, a quantity that varies (*variable*), or every number (*the general case*).	'Solving' an equation consists of determining which value(s) for a given symbol make(s) the equation true.
		An equals sign (=) conveys equality, whereas ≠, < and > convey inequality.	Real-life situations and functions can be represented in a variety of forms, including numbers, words, symbols and tables.	When generating an expression to represent a real-life situation, it can be possible and useful to 'simplify' a long or complex expression.
		In number sentences (*equations*), symbols can stand for something we need to find out, or they can express a relationship.	A function is a special relationship where each input has an output. There are always three main parts; the input, the functional relationship and the output.	An expression may contain more than one unknown or variable. Each unknown or variable must be represented by a dedicated symbol.
		In a number sentence, number facts can be applied to help find an unknown value.	Representing the structure of a function using words, symbols, graphs, tables or diagrams is useful to identify outputs for given inputs and vice versa.	A real-life situation can be represented by an expression or a series of expressions.

Data				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Objects and sets can be sorted according to one or more attribute.	Investigations are cyclical and are motivated by posing a question.	Data displays can hold a vast volume of information which can be reasoned and from which dedications and inferences can be made.	The mean, median and/ or mode are measures of centres which communicate different centres of the data and provide a range of insights.
	A data set is a collection which holds specific attributes or information.	Data investigations involve a process of planning, collection, gathering, representation and analysis of data, and communicating conclusions that answer the question.	Data displays are selected and justified based on their ability to communicate aspects of the data and answer the questions posed. Moving between data displays allows for further comparison and analysis.	Samples can be drawn from a population of data as representative evidence, to make generalisations and determine the degree of confidence or certainty about the generalisation.
	Data is all around us and helps us interpret the world.	Data can be qualitative (it describes something) or quantitative (it holds numerical value).	Measures of centre (e.g. the mode, median or mean) are one number summaries of entire distributions.	Reported data can be evaluated in terms of its representativeness, intentionality and reliability.
	Data displays are a useful way of conveying information.	Different types of data require different graphs and different statistical measures.	The range is a measure used to capture variability or spread of the data.	
	Data can be collected and represented in many ways.	Graphs are tools which communicate distribution, centre and variability of data.	Secondary data can be analysed to make observations or inferences and to draw logical conclusions.	
			Informal inference is about moving beyond the data collected (sample) to a wider context (population).	
			Data can be distributed in different ways. Such distributions of data can be compared according to their shape.	

Chance				
			Stage 3 (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts			Events in everyday life involve chance. Some events are more likely to happen than others.	Probability can be represented on a scale between 0 – 1.
			If an event is unlikely to happen, it has a low probability. If something is likely to happen, it has a high probability.	The experimental probability of an event occurring may not always match the theoretical probability.
			Expected or predicted outcomes of an event can differ from actual outcomes.	The probability that a specific outcome will occur can be represented as a fraction, decimal or percentage.
			Investigating chance allows decision-making and predictions about everyday events and occurrences.	A sample space contains all possible outcomes of an experiment.
				Probability can be described in proportional terms and is calculated by dividing the number of ways the identified outcome can happen over the total number of possible outcomes.
				The greater the number of trials brings the actual outcomes closer to the expected outcomes.

Measuring				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Objects have attributes that can be measured such as length, weight, capacity and area.	Common base units of measurement are useful to make and test comparisons.	Metric units help us to interpret, communicate and calculate measurements with increasing accuracy and precision.	Purpose and practicality are important to consider when measuring attributes and selecting units and instruments for measuring.
	The purpose of measuring is to compare.	The size (metric) of the unit affects the number of units needed to measure an object.	Measurements can be made more precise by selecting metric units (multiples or subdivisions of base units e.g., km or cm).	Purposeful descriptions and comparisons often involve the measurement of more than one attribute.
	We can compare and order things by how much of a particular attribute (physical quantity) they have relative to each other.	We can compare, measure and order physical quantities by selecting the appropriate unit and determining how many units the thing has/holds.	The relationships between metric pre-fixes can be understood and applied in a similar way across different units of measurement.	The relationship between equivalent units in the metric system help us to judge attributes, move flexibly between units and do calculations.
	Attributes are compared and ordered using units of measurement.	Measurement instruments (e.g., rulers) are tools for measuring physical quantities or attributes such as length, weight and capacity.	The metric system is based on multiples of ten. Any measurement given in one metric unit (e.g., kilogram) can be converted to and renamed as another metric unit (e.g., gram).	Measurement sense develops as we anchor the meaning of measurement units to measurement benchmarks in the everyday world.

Time				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Time is temporal and passes consistently.	Time is measured using universal units; seconds, minutes, hours, days, weeks, months, years and centuries etc. There are distinct relations between these units.	60 is the base of the number system for measurement for hours, minutes and seconds and can be expressed in fractional terms (of 60).	Greenwich Mean Time is used as the standard time against which all the other time zones in the world are referenced.
	Events in daily routines can be described and sequenced. A timetable is a useful display to show when things will happen.	Units of time measure how long something lasts.	Time is expressed in relation to the hour using minutes or fractional intervals.	Speed is measured as distance travelled per unit of time.
	Time can be expressed and recorded in a variety of ways.	The hour and minute hands of the analogue clock move clockwise as time passes.	Time can be represented using 12-hour or 24-hour formats. The 24-hour format shows the number of hours and minutes elapsed since midnight.	
	Each hour has an area or space on the analogue clock.	Time can be represented in both analogue and digital formats.	Timetables and schedules are tools for managing and organising time.	
	Calendars displays day and month and can be used to highlight times of significance.			

Money				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Money comes in many forms.	The price of an item or service is determined by its relative value.	Understanding the unit price helps us make judgements about value for money.	Budgets are useful tools for organising, managing and recording money and transactions.
	Money is used in the buying and selling of goods and services.	Set amounts of money can be represented by different combinations of coins and/ or notes.	Calculations of money can be approximated and determined by performing operations.	Currency is the medium of exchange of money (notes and coins) in common use in a nation. The currency exchange rate is determined by the value of the currency and is used to convert the value or quantity of one currency into the relative value or quantity of another.
	Goods or services can be exchanged for a set amount of money.	Monetary transactions can be recorded as number sentences.	Transactions and calculations of money can be recorded in different ways.	Tax is a contribution to state revenue. It can be deducted from income or business profits or added to the cost of goods, services and transactions.
	The units of money used in Ireland are euro (€) and cent (c).	The symbols of € (euro) and c (cent) are used to express and record money.	Money is recorded in decimal form.	Interest describes how much is paid for the use of money.
	Monetary transactions happen in a number of ways.	Various strategies can be used to calculate change in a transaction.	Estimation and rounding can help judge the reasonableness of transactions.	

Uses of Number

	Stage 1 (Junior & Senior Infants)			
Learning Outcomes				
Mathematical concepts	Numbers can be used in different ways.			
	Numbers are used for labelling and identification purposes (nominality).			
	Numbers denote quantity or the amount within a set (cardinality).			
	Numbers describe the position of something in a list such as 1st, 2nd, 3rd (ordinality).			

Numeration and Counting

	Stage 1 (Junior & Senior Infants)	Stage 2 (1st & 2nd Class)		
Learning Outcomes				
Mathematical concepts	Quantities can be subitised and compared without needing to be counted or assigned a numerical value.	Estimation and counting strategies can be applied to determine quantities / calculations.		
	There are five principles of counting; one-one, stable order, cardinal, order irrelevance and abstraction.	The reasonableness of estimations can be tested by counting.		
	The last number in the count indicates the quantity in a set.	There are a range of strategies for counting forwards and backwards.		
	Quantities in a set do not change with physical rearrangement of the set.	Patterns exist in number-word counting sequences, within the base ten number system.		

Place Value and Base Ten				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Numbers can be distinguished according to their quantitative value.	The value of a digit in a number depends on its place. The position of a digit denotes a value ten times that of the digit to its right.	The value of a multi-digit number is represented by the value of each of its constituent digits.	Fractions, decimals and percentages are three ways of expressing part-whole relationships.
	The base of our number system is ten.	A new place value unit is formed when ten of the previous place value units are grouped.	The principle of base ten holds for all numbers, including whole numbers and decimals.	Rationale numbers can be expressed as fractions with a denominator that is a power of 10.
	The base-ten number system consists of 10 digits and is based on groups of ten.	The relationship between one quantity and another quantity can be an equality or inequality relation.	A decimal point is a convention that separates whole numbers (left) from parts of a whole number (right).	Multiples of 10 are a useful tool for converting between fractions, decimals and percentages.
	In a 2-digit number, the digit to the left denotes the greater value.	0 can be used as a placeholder, allowing us to record a number accurately.	The base ten place value system extends indefinitely in two directions from “one”.	A percentage is a way of expressing a fraction of one hundred or another way of writing hundredth. Per ‘cent’ means out of a hundred and uses the % notation.
		Numbers can be rounded or approximated to provide estimations of value.		

Sets and Operations				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Quantities (or sets) can be partitioned and combined.	Commutative, associative, additive identity and distributive are significant properties of addition.	Commutative, associative, additive identity and distributive properties also apply to the operation of multiplication.	Estimation and rounding are useful to test the reasonableness of answers to more complex operations.
	Adding to a number makes the number (quantity) bigger. Subtracting from a number makes the number (quantity) smaller. This can be represented as a move on the number line or 100 square.	Numbers and symbols are used to construct and express number sentences. These can help to solve problems or are used to express contexts mathematically.	Multiplication is defined as having a certain number of groups of the same size. An early representation of multiplication is repeated addition.	For fractional and decimal computation, the ideas developed for whole-number computation sometimes do not apply.
	A number does not change when adding or subtracting zero from that number.	When combining or partitioning numbers, we sometimes need to exchange tens to units, or hundreds to tens where necessary.	The conventions for performing operations on whole numbers are the same for decimal numbers.	A prime number has exactly two factors – itself and one, a composite number has three or more factors. The number one is neither prime nor composite.
	Addition and subtraction have an inverse relationship.	A number fact is a mental picture of the relationship between a number and the parts that combine to make it.	Division can be described as the splitting of a number into equal parts or groups, or the repeated subtraction of a number.	Factors are numbers that multiply together to give a product.
		Representations of subtraction can include reduction, complement and difference.	Multiplication and division have an inverse relationship.	Multiples are the result of multiplying a number by a whole number.
			Use of a calculator can reduce computative focus allowing for increased focus on strategies.	

Fractions				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Sets, objects and spaces can be partitioned in different ways.	Each equal share of a set has the same value.	A numerator denotes the number of parts, the denominator denotes the total number of parts in a whole.	Fractions can be more easily added / subtracted when they have a common denominator.
	Fractions are a representation of part-whole relationships.	Numbers may be expressed as numerous equivalent fractions.	A fraction may be considered as a representation of division.	Fractions can be represented in decimal and percentage form.
	Fractions are named according to their number of equal parts or shares.	The greater the number of portions of a whole, the smaller the size of each portion.	Fraction families are helpful to show how fractions are related and / equivalence, and when adding and subtracting fractions.	Ratios can be used to compare two or more whole numbers and have corresponding representations as fractions.
			Fractions can express value greater than one. Improper fractions have numerators that are higher than the denominators.	Multiplying or dividing a fraction by a fractional equivalent of one does not alter its value. This can be useful for exploring equivalence and / or computation involving fractions.

Spatial Awareness and Location				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	Spatial awareness involves the awareness of oneself in the spatial environment.	Knowledge of one's location in the environment is necessary for successful orientation, way-finding and navigation.	Lines can be classified depending on their orientation and their interaction with one another.	Location can be described using co-ordinates.
	Simple maps and/ or routes can be used to track the movement of objects.	When two straight lines intersect, an angle is formed at the point of intersection.	Angles can be classified according to their size.	The sum of interior angles of a shape is determined by the number of its sides.
	Position can be viewed from various vantage points.	Turns can be described in terms of direction and extent of turn.		Co-ordinates can be plotted on a cartesian co-ordinate plane.
	Language can be used to describe how objects and people fit and move in relation to one another.	The location of objects can be portrayed on a grid system.		

Shape				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	3-D and 2-D shapes have different names depending on the properties and distinct features they possess.	3-D and 2-D shapes can be identified, categorised and distinguished by their properties and rules.	Shapes and shape families can be sorted and classified according to multiple properties and rules.	3-D and 2-D shapes can be measured and tested for the constituent properties and rules.
	3-D and 2-D shapes can be sorted and discriminated according to their shape attributes.	Shape families contain shapes that have common properties or features.	A net is a representation of a 3-D shape which can be folded or assembled to re-create the 3-D shape.	Given some information about lines and angles, measurements can be deduced and used for construction.
	2-D shapes are faces or flat representations of 3-D objects.	Shape families have a range of shapes within them.	Properties, rules and measurements of a shape can be investigated by construction, deconstruction and dissection.	To construct models or structures using geometric shapes certain rules must be followed.
	Models and structures are made up of different shapes.	A polygon is any 2-D shape with straight sides. The name tells you how many sides the shape has. To be a regular polygon all the sides and angles must be the same.	Prisms and pyramids gain their names from their polygon bases.	

Transformation				
	Stage 1 (Junior & Senior Infants)	**Stage 2** (1st & 2nd Class)	**Stage 3** (3rd & 4th Class)	**Stage 4** (5th & 6th Class)
Learning Outcomes				
Mathematical concepts	A shapes position, orientation or size can be changed without changing the kind of shape it is.	A shape or line is reflected when it is the same perpendicular distance from the mirror line.	Certain shapes and combinations of shapes can tesselate.	Transformations involve a number of steps that can be recorded, tested and performed.
	Shapes can be combined to make structures.	A shape or line is rotated when it is turned around a point called the centre of rotation.	Co-ordinates are numbers that determine the position of a point or a shape in a particular space (on a map or graph).	When shapes are transformed their co-ordinates can be predicted and deduced.
		A shape or line is dilated when its size changes (gets larger or smaller), but its position remains the same.	Cartesian co-ordinates are pairs of numbers, the first of which indicates the point on the x-axis and the second on the y-axis.	
		A shape or line is translated when it is moved a certain distance from its original position (without turning).	When shapes are transformed on a plane their co-ordinates change (and can form patterns).	

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