

Consultation Report on the Background Paper and
Brief for the Review of Applied Mathematics

Contents

Consultation Report on the Background Paper and Brief for the Review of Applied Mathematics	1
1.Introduction	5
1.1 Focus Group meetings	5
1.2 Online questionnaire	5
1.3 Written Submissions	6
1.4 Focus of the consultation	6
2. Feedback from the focus groups	7
2.1 Concerns about current Applied Mathematics education	8
2.2 The purpose of mathematics education at second level	11
2.3 Expanding the specification beyond mechanics	13
2.4 Aim of the new specification	14
2.5 The purpose of the specification and its likelihood to broaden the appeal	14
2.6. Implications for Implementation	18
2.7 Assessing Applied Mathematics in the twenty-first century	20
2.8 The role of technology	20
3. Feedback from the online questionnaire	23
3.1 Purpose of senior cycle mathematics education	23
3.2 Role of Problem Solving	24
3.3 Conceptualisation of mathematics teaching and learning	24
3.5 Perspectives on assessment	26
3.6 Perspectives on technology	26
4. Conclusion	27
Brief	28
Appendix 1	30
Focus group questions	30
Appendix 2	33
Appendix 3	34

1. Introduction

The Background Paper for the Review of Applied Mathematics was approved for consultation by Council in March 2014. The NCCA consultation process ran from November 3rd 2014 to December 19th 2014 and consisted of the following elements:

- an online questionnaire
- focus group meetings with stakeholders

The aim of this process was to hear the views of all stakeholders and the general public.

1.1 Focus Group meetings

A number of focus group meetings were held with stakeholders, as follows

- Irish Applied Mathematics Teachers Association (IAMTA) AGM in UCG, November 29th 2014
- Mathematics and Applied Mathematics Department personnel in
 - University College Dublin, December 15th 2014,
 - Trinity College Dublin, December 17th 2014
 - University of Limerick, January 19th 2015
- Mathematics Education Department (students and lecturers) in
 - Maynooth University, Monday December 8th 2014
 - Trinity College Dublin, Tuesday December 9th 2014.

See Appendix 1 for the focus group questions.

1.2 Online questionnaire

A consultation page was created on the NCCA website. Here respondents could access the *Background Paper for the Review of Applied Mathematics* and complete the online questionnaire (Appendix 3). The consultation process remained open for 7 weeks beginning on November 3rd 2014 and was publicised through the website and social media.

1.3 Written submissions

While there was no specific call for written submissions a number were received from (Appendix 2)

- IAMTA
- IMTA
- UCD Mathematics Department

There were also four submissions from individuals.

1.4 Focus of the consultation

The focus of the consultation process on the following aspects of Applied Mathematics was similar for both the consultation meetings and the online survey:

- Review of the current experience of applied mathematics education
- The purpose of mathematics education at senior cycle
- Students' engagement with senior cycle mathematics
- Readiness for post-school pathways
- The role of problem solving in the new specification
- Broadening the appeal through the new specification
- Possible frameworks for the new specification
- Perspectives on assessment and the role of technology.

2. Feedback from the focus groups

While a broad range of participants attended the focus group sessions, the majority were involved with the current specification at some level. Teachers who currently teach Applied Mathematics were well-represented at focus group sessions as were third level lecturers in mathematics and Applied Mathematics. A large number of the student teachers that attended the focus group sessions had studied Applied Mathematics inschool.

Participants were asked about their initial impression, having read the *Background Paper for the Review of Applied Mathematics*. There was a wide range of responses ranging from concern and scepticism through to a tentatively positive and welcoming response.

The concerns related in part to the use of quotations from the paper, with some respondents of the view that these were used out of context in a biased and selective way to show the current Applied Mathematics syllabus in a negative light.

Practitioners of the subject quoted in a selective way to cast subject in a dim light.

Quotes chosen to suit paper A selective and biased to show Applied Maths is not problem solving.

Many respondents were highly sceptical of the rationale for the proposed changes and comments reflected the view that the paper and proposed changes are strategic and linked to compensating for the changes introduced by the *Project Maths initiative*.

A lot of what is in the core is there for political reasons because it was left out of Project Maths and this was seen as a deficit and will be lumped into the new spec to say it is there.

There were a number of fears outlined including the fear that the background paper was suggesting a change in the focus of the subject Applied Mathematics which might have a detrimental effect on core skills such as problem solving.

Including more content could lose focus on skills.

Respondents who were critical of the background paper claimed it had been overly influenced by the needs of third level and industry and that it was prepared in response to PISA.

The background paper is quite critical of Applied Maths teachers and the current syllabus, it seems to be heavily influenced by third level interests.

Education for what economy thinks it needs would be wrong.

PISA results drive change.

Within the initial responses, even amongst those that were positive, there was concern to avoid any tendency for teachers to omit parts of the course and simply teach to the examination.

The proposals are absolutely brilliant, taking Applied Maths out to other topics but concerned about the 'teach to the test' mentality, don't want to end up with lovely stuff but not all of it getting taught.

2.1 Concerns about current Applied Mathematics education

Participants were asked their views on how well the background paper had dealt with concerns around the current provision for Applied Mathematics, what the challenges of dealing with these concerns might be, and how these might be dealt with.

A lot of discussion focused on the view that Applied Mathematics is undervalued in schools, as evidenced by the fact that it is not always available on the timetable and is seen as a specialist subject, and even elitist.

Not offered in my school. No talk of it at all, never brought up as an option.

It's not being picked up because it's difficult for students and difficult to teach. There is kudos for people who do it.

Lots of people are of the view that if you do Applied Mathematics you will do well in it. It's a specialist subject.

Applied Mathematics involves a small group of high achievers.

Some participants highlighted the fact that as a course it had remained unchanged for many years.

Exam papers from the '80's still ask the same questions, textbook published in '78.

There was a strong response to the claim in the background paper that Applied Mathematics is a predictable subject with many respondents strongly disagreeing with this view.

Whilst there may be some predictability about the topic in a given question, the questions themselves are not predictable, students will have to work out how to solve them.

However other respondents pointed out that they wondered whether students would choose Applied Mathematics if there wasn't such close alignment between the examination and the classroom experience.

There is no doubt about it, you can train people to do well in Applied Mathematics because the exam is so predictable. They go to class for an hour a week, practice like mad on questions and the one they get in the exam won't be much different. If it is, they can't cope. I doubt people would choose it if that changed.

A second area of disagreement with the background paper was with the claim that the overemphasis on content may be responsible for the narrow appeal of Applied Mathematics. A number of explanations were offered for the narrow appeal. These are quite varied and often conflicting, for instance some claim that Applied Mathematics is chosen because of the potential to get an A grade.

Taken as the 8th subject outside class. If the % of A's drop it has a significant impact on the number taking it the next year.

If it's not seen as doable and getting points, it's dropped.

Whilst others felt an A grade wasn't easy to achieve.

It's not true that Applied Maths is an easy grade. They have to work but they enjoy it.

The fact that maths in general is seen as difficult and that there is a fear amongst students about maths was emphasised as being one of the reasons for low uptake.

Students think it is too hard and choose the path of least resistance.

Low numbers is about proficiency of students coming from Junior Cert. Students with an A at JC are not always robust enough to deal with Applied Mathematics.

It's acceptable to say you're lousy at maths in Ireland, not so in other EU countries.

The impact of timetabling and the availability of both time and teachers within schools was highlighted as a significant reason for the narrow appeal.

Applied mathematics is not offered in large parts of the country. One school in the whole of Tipperary offers Applied Mathematics.

If Applied Mathematics becomes a full timetabled subject, this will give rise to issues for students (in terms of choice) and schools (in terms of options).

The role of career guidance was highlighted, the view being that students are not always told what professions Applied Mathematics would be useful for. Participants emphasised the important role that career guidance teachers have in pointing out the relevance of Applied Mathematics to specific careers and that this will need to be improved to increase the appeal of the subject to students.

Career guidance is very important - they never tell kids who want to study engineering in IT's to take Applied Mathematics.

Students make choices based on points, not on what they want to do after school.

There was a concern that any proposed change to the current Applied Mathematics curriculum could have a negative impact on problem solving. The view was strongly held that Applied Mathematics is a problem solving course and one of the few curriculum areas of its kind where students can gain these skills. Participants emphasised strongly the importance of avoiding any dilution of the problem solving aspects of Applied Mathematics and there was some concern that expanding the content away from mechanics might impact negatively on problem solving.

Applied Mathematics is essentially problem solving through the lens of mechanics.

If we bring in other content we may lose the problem solving skills

A number of contradictory views were expressed around the impact of *Project Maths* on Applied Mathematics. These ranged from a belief that the omission of a number of topics from the *Project Maths* syllabus has impacted negatively on students' mathematical skills to the opposing view that *Project Maths* has had a positive impact on the take up of Applied Mathematics and maths skills in general.

Algebra skills are very poor because of Project Maths.

The impact of Project Maths is increased uptake of Applied Maths, possibly because of the emphasis on problem solving in the Project Maths syllabus.

A number of challenges for teaching Applied Mathematics were identified. These include students' ability, student learning styles, the perception of Applied Mathematics as being difficult and finally how problem solving is managed within the Leaving Certificate in general.

Students are afraid of a question they haven't seen before, afraid to think outside the box because they haven't done it in other subjects.

The level of maths now at JC and LC is lower.

The negative impact of exams and the CAO system was also highlighted.

The focus is on exams and students don't think of the benefits of what they learn.

Any kids I know do it because they'll get high points.....they know they're good at it and they do Physics and Maths too.

2.2 The purpose of mathematics education at second level

The Background Paper for the Review of Applied Mathematics presented possible pathways that students can take through mathematics at senior cycle. Participants were asked their views on the purpose of mathematics education at senior cycle and the benefits and challenges of this proposed pathways approach. One of the key points raised in response to this question was that the proposed changes would lead to a subject that would not be Applied Mathematics but rather Advanced Mathematics. Whilst there was welcome for the adoption of a pathways approach to mathematics education in Ireland there was concern that should this approach be adopted, students who continue to view Applied Mathematics as a minority subject and would not select it. This will further disadvantage those students when it comes to making third level choices. This will become a real issue should the new specification become a requirement for certain courses in university; further evidence of the importance of the role of career guidance in promoting this new specification.

To deal with the restricted amount of maths in Project Maths, advanced maths needs to be introduced.

I really like the idea of double maths but most fifth years don't know what they want to do so they might not take it and then won't get into the course they decide to do.

Much of the concern was related to a number of key beliefs, firstly the existence of a dichotomy between mathematical knowledge related to the content aspects of the syllabus versus problem solving which is seen as a skill. A strong view emerges throughout the consultation that adding content to the Applied Mathematics syllabus will dilute the development of problem-solving skills.

If you compare the mathematical content of maths and Applied Maths the crossover is [very little] sin, cos and tan. There is very little crossover, you need basic logical reasoning after that.

If you put all that core [vectors, matrices, differential equations] into Applied Maths then the subject is no longer applied maths.

Applied Maths is problem solving. They enjoy that... it's the maths that is holding them back.

Secondly those involved in teaching Applied Mathematics do not believe that they should have to teach mathematics. Instead they see their focus as being on teaching problem solving skills.

We don't teach maths. If you put it in the core and make us teach it we'll have no time for problem solving...Applied Maths is problemsolving.

One view of Applied Mathematics is that the subject draws upon concepts and methods of mathematics from the fields of application and in turn, brings ideas, techniques and scientific knowledge back to influence the development of mathematics. Adopting this perspective would mean that this is not an either or situation but rather mathematical knowledge and problem solving skills are complementary aspects of mathematical understanding required to solve real world problems in the twenty-first century.

This apparent dichotomy may be a legacy of the previous approach to mathematics teaching and learning prior to the *Project Maths* initiative; students learned knowledge in the mathematics and physics classrooms and this knowledge was built upon and used in the Applied Mathematics classroom in a problem solving way.

Where students are pursuing their further studies in engineering within institutes of technology it is important that they get the opportunity at second level to develop their problem solving skills. Currently some of these students opt for Ordinary Level Applied Mathematics and there is concern that this cohort may miss this opportunity to develop problem-solving skills if Applied Mathematics becomes further mathematics.

I would like to see this problem solving option open to all students not just those high flyers.

A number of comments reflected how subject choices are made at second level. Not everybody has the opportunity to choose Applied Mathematics even though their future careers and third level studies might benefit from this.

We were never told about Applied Maths or how useful it would be. It just was about what option streams do you want.

Some comments were made about the challenge of studying STEM subjects at third level when the student has not had the opportunity to study Applied Mathematics at second level. Sometimes it is because it is not available and sometimes it is down to a lack of awareness of their desired future pathway, further highlighting the importance of career guidance early in second level and engaging students in discussion of possible future aspirations.

You don't take it because you need it, you take it because you like it and you're good at it and it gets you points.

I knew I wanted to go the maths route in College but to be honest I wouldn't have taken it if it was hard to get an A. I liked it but at the end of the day I had 8 subjects.

2.3 Expanding the specification beyond mechanics

Participants were asked about expanding the specification beyond mechanics to support a pathways approach. A provision of different 'curriculum pathways' which allow students to choose mathematics courses that more closely match their future mathematical needs

In responding to this question it became very clear that there is a perception that this review threatens the problem solving content of Applied Mathematics and is being used to solve other problems in mathematics education.

Most sixth years don't know what they want to do and if they go to college to do engineering without an extra pathway then they will be disadvantaged further.

While there was a broad welcome for the idea of a pathways approach the challenges of timetabling were highlighted.

How would this be put on the timetable? Would it be seven subjects or eight? What would it be offered against?

Support was voiced for expanding the content beyond mechanics with participants noting that the current mechanics content is nineteenth-century mechanics which is not motivating for many.

Even within mechanics we could move out of the nineteenth-century and into at least the twentieth century. There are much more interesting modern examples we could study.

In the current Leaving Certificate structure students confuse Applied Mathematics with Mechanics and it is difficult to disabuse students of this notion on arrival at university.

Make it interesting. There are so many interesting applications of mathematics that would motivate students and increase the appeal of this subject. It can't be just mechanics!

2.4 Aim of the new specification

The background paper proposed that the aim of the specification should be *to develop problem solving skills in Mathematics which can be transferred to unique and novel situations to which students have not yet been exposed* and respondents were asked for their views on this aim. The responses in this section focused on participants' views of the current specification. It was again made very clear that Applied Mathematics is viewed by many as a problem solving course already and that there is a concern amongst participants now that adding content to the syllabus would dilute the problem solving aspects of the current course.

2.5 The purpose of the specification and its likelihood to broaden the appeal

Once again responses to this question revealed the recurring theme of *What is Applied Mathematics?* The perception of a trade-off between content and developing problem solving skills was again in evidence in the responses. Participants clearly expressed the view that Applied Mathematics in its current format is a problem solving subject where students develop problem solving skills through the lens of mechanics.

The current Applied Mathematics is the only place where students are not given answers to rote learn. They do apply critical thinking.

Mechanics is a laboratory where students get to explore and build their experience in reasoning.

Three possibilities on proposed structures for the new specification were outlined. Participants were strongly of the view that whatever is put in place needs to be of a high quality and should not lose the problem solving skills aspect. Possibilities 1 and 2 propose a core and options structure as set out below. Possibility 3 is slightly different. It abandons the core and options structure altogether and simply provides curriculum pathways representing a number of specialised areas where mathematics is applied. This provides a structure for students to combine their interest in mathematics and

mathematical reasoning, with an interest in a specific field of intellectual activity.

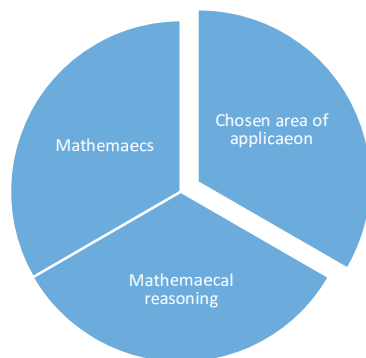
Possibility 1

	Units	Weighting
<i>Core</i>	<ul style="list-style-type: none"> – Matrices and their applications – Vectors and their applications – Linear programming – Further calculus – Differential equations 	70% 125 hours
<i>Options</i>	Mechanics	30% 45 hours
	Computer programming	30% 45 hours
	Networks and graph theory	30% 45 hours
	Business mathematics	30% 45 hours
	Game theory	30% 45 hours

Possibility 2

	Units	Weighting
<i>Core</i>	<ul style="list-style-type: none"> – Matrices and their applications – Vectors and their applications – Linear programming – Further calculus – Differential equations 	40% 80 hours
<i>Options</i>	Mechanics 1	30% 45 hours
	Mechanics 2	30% 45 hours
	Computer programming	30% 45 hours
	Networks and graph theory	30% 45 hours
	Business mathematics	30% 45 hours

Possibility 3



Possible areas of application include
Physics, Chemistry, Computer
Science, Business, Social Sciences,
Medical Sciences and Engineering.

In relation to the first two possibilities, challenges were highlighted in timetabling, teacher skills and resources. The fear that the choice would be teacher driven rather than student driven also emerged as a concern.

Can't see where you'd arrive at a situation where students have individual choices in class. You [students] can have choice in exam but realistically it will be teachers' choice [in the class] and they [teachers] will all choose mechanics.

The challenges of a student driven approach when there is low uptake were highlighted

A teacher could have five students all sitting around doing different options. How would that work?

It was also noted that if you have options and you can only choose one or two this might be limiting, as all might be useful going forward.

I welcome the attempt to diversify the "options"; however, it is these I feel should be core. The shame is that they can take just one or two of these. I think a syllabus where students can select four out of seven or eight applications would ensure the diversity that ought to be aspired to.

It was felt that the first possibility would be more detrimental to the development of problem solving skills and respondents felt that this possibility represented an Advanced Mathematics course

as opposed to Applied Mathematics. It was suggested by some that this was actually six different subjects rather than one coherent one.

Possibility 1 is worse for loss of problem solving skills, more maths really Further Maths not Applied Maths.

In some cases, respondents felt the second possibility was preferable as it was the closest to the current situation.

I'll choose possibility 2 but there will be no change. With possibility 2 everyone will do the same as they do now. It won't widen the appeal because mechanics will be the only option.

Others felt the second possibility was the less preferable because the additional time required to deal with the extra mathematics content in the core might impact negatively on the problem solving aspect.

Don't see the capability of teaching problem solving through vehicle of the core, through options, yes.

It was pointed out that if a core and options structure were to be adopted then one of the major challenges would be the weighting of the options.

How could you decide which options were harder than others? It would cause a problem with genuine choice. You know how exam focused everyone is.

In relation to computer programming it was strongly felt that there was great benefit in having this at Leaving Certificate level but not as part of a mathematics course.

There is no computer science at LC and this is a great shame but it is a subject in its own right and shouldn't be lumped into Applied Maths.

If the inclusion here of Computer Programming is aimed to placate the calls for the proper introduction of Computer Science as a full subject at Leaving Certificate, then absolutely no, not nearly good enough. If the inclusion here of Computer Programming is a pathway to the introduction of Computer Science as a full subject at Leaving Certificate then fairenough.

An interesting modification of the options structure was suggested in a submission

Rather than offering a finite and static list of options likely to lead students to the mistaken conclusion that there is a finite list of canonical topic areas comprising

the whole of Applied Mathematics, a number of case studies in application areas would be made available to students in any given year. From this list, students could choose five case studies and spend nine hours on each.

The idea suggested here is that the core and options structure be replaced with a core and case study structure and the mathematics in the core be introduced to motivate the case studies

In deciding what potential topics would be included in a new core syllabus a key criterion would be to select those topics that would feed into the case studies in a natural and unforced way.

It was also suggested that rather than include computer programming as a potential option that this idea should be *modified* to include *computational sciences* since

Computational science; the formulation and implementation of numerical algorithms to solve equations arising from mathematical modelling in the sciences is at the heart of modern applications of mathematics

When it came to discussing the merits of the third possibility some interest was expressed, although respondents felt that more detail was needed to flesh out the mathematical application aspect. Participants also highlighted some of the benefits.

Like this, but don't have clear picture of where it's going, not fleshed out enough. Interested but scary.

This would allow for interesting problems motivated by what the mathematics students already know.

2.6. Implications for Implementation

Regardless of the possibilities chosen, there was a consensus that there are major implications for teacher support, teacher capacity, teacher knowledge, in-service and pre-service education. There is also recognition that the nature of these changes would have an impact on the role of the teacher. Some mentioned this in a positive light, that it would involve teachers encouraging students to be more mature and responsible for their own learning, which was seen as very necessary for a smooth transition from second to third level. The whole idea of mentoring arose and participants recognised the need to change how they teach and clearly identified the necessary supports required to facilitate this changed role.

Different sort of teaching is required, self-directed learning with a focus on skills.

Encouraging students to be more mature and responsible is good. They learn with mentorship.

There was a definite acknowledgement that this subject needed full timetable status and the need for innovative whole school solutions to the challenges this would create were noted, thus highlighting the need for management support and input into the development of this specification.

Principals will need to be innovative and put their thinking caps on to deal with this one

The question of how ready the current cohort of both teachers and students would be to take on this subject, regardless of which possibility is chosen, was also raised.

How confident is the current cohort of students to take the subject? We know that all third level institutions are putting on remedial courses. How will they [students] cope? Will they [students] have the skills?

Anyone who did maths in college over the last ten years will have no problem with all those options but what about the ones who teach applied maths now?

Issues regarding the naming, branding and selling of the subject were also raised at this stage.

The name of the subject is important. You need to forefront problem solving. Computational Problem-Solving?

The need for more detail and the whole idea of dovetailing it with *Project Maths* were noted and a warning was given not to rush the developments.

Clearly has to link to Project Maths syllabus....has to dovetail to Project Maths.

It could be done on a phased basis (as was Project Maths) and with a long lead-in time. I think the objective [increase uptake and motivate students] will be well worth it in the end.

Another issue, which began to be identified at this stage, was that of marketing and trying to attract students to undertake the new course. Respondents noted the importance of making the specification exciting, interesting and relevant for students and teachers alike.

This presents an exciting opportunity and if handled correctly has the potential to transform mathematics education in a very positive way. We have the opportunity here to hook kids on maths.

2.7 Assessing Applied Mathematics in the twenty-first century

There was recognition that there is a limit to what can be examined in a written examination, however there is a consensus that alternatives will pose their own challenges. Some suggestions were made to move the assessment away from a final exam, including a viva, a project, a case study and modular examinations taken intermittently throughout the year. All these suggestions highlighted the need for robust and valid assessment.

2.8 The role of technology

There was recognition that there is the potential to use technology in a positive way in Applied Mathematics, though the point was also made that it should not be technology use for technology sake and should not compromise the development of skills but rather support and facilitate them. Many benefits of the use of technology were noted

There is a huge opportunity to deliver this course using technology this would overcome many of the problems around choice and teacher availability.

Technology if suitably mediated by a teacher will help both kids and teachers ..they'll [teachers] change their role and kids can be motivated by enquiry.

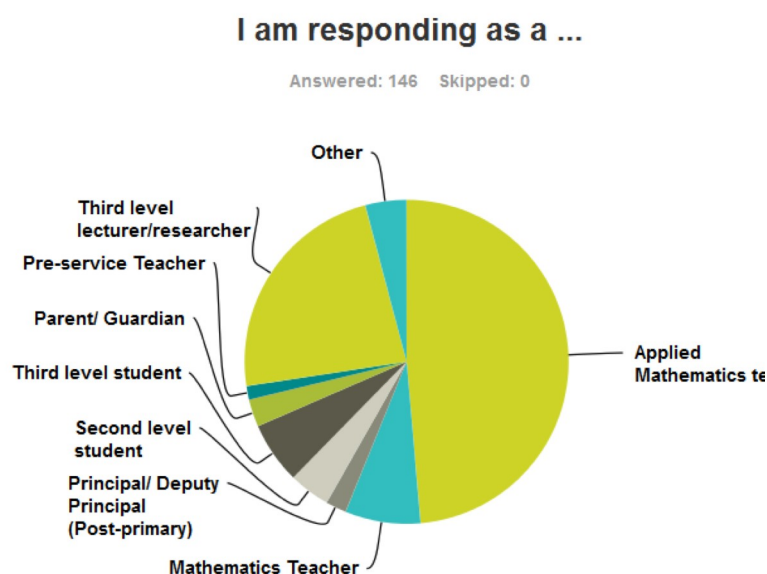
Must get students in 2nd level using computer algebra systems.

Summary

- There is support for a pathways approach to mathematics education but the new specification should not be a Further Mathematics course. There is consensus that the focus of the new specification should be on the development of problem solving skills through the application of mathematics to solve real world twenty-first century problems in motivating and engaging contexts.
- There is support to expand the areas of application in Applied Mathematics beyond mechanics.
- There is consensus that the inclusion of content should not impact negatively on the development of problem solving skills and perhaps should be considered only as part of a major and direct application.
- There is recognition that the context from which the real world problems are chosen has the potential to motivate and engage students as well as broaden the appeal of the subject.
- There is consensus that the specification should be prepared for 180 hours of study.
- There is recognition that technology has a role to play and its potential for overcoming some of the challenges highlighted should be explored
 - Timetabling
 - Teacher support and availability
 - Development of problem solving skills.
- Perhaps a core and case study structure may be more suitable than a core and options structure.
- Perhaps computational science would be more suitable than computer programming for inclusion in the specification.
- There is support for the introduction of alternative modes of assessment provided they are robust and valid and have the support of teachers.

3. Feedback from the online questionnaire

In total, 146 respondents completed the online questionnaire (see Appendix 1) with almost 50% identifying themselves as post-primary applied mathematics teachers and just under 25% identifying themselves as third level lecturers/researchers.

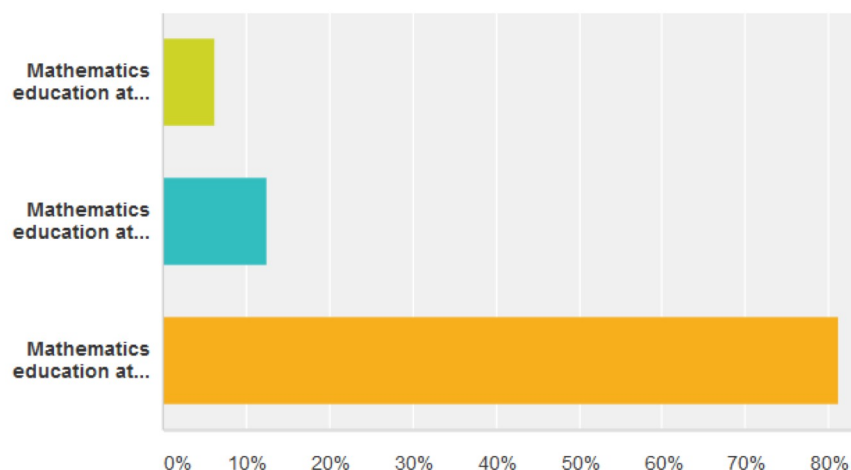


An analysis of the open responses throws up three main themes consistent with the focus group responses; problem solving, content and timetabling.

Highlights from the quantitative analysis are outlined below under six main headings.

3.1 Purpose of senior cycle mathematics education

There is a consensus around the purpose of senior cycle mathematics education with over 80% of respondents agreeing that *mathematics education at senior cycle should prepare students to participate fully in modern society and at the same time prepare them for further study in mathematics related disciplines.*



Consensus was also reached in relation to the time available to students to engage with mathematics at senior cycle with 92% of respondents agreeing with the statement that *students who wish to pursue further study in mathematics related disciplines should have the opportunity in school to engage with mathematics for longer than the 180 hours allocated to the mathematics specification.*

3.2 Role of problem solving

There is support for the notion of a change in relation to the role of problem solving. While 46% of respondents believe that the current Applied Mathematics syllabus develops problem solving skills which can be transferred to unique and novel situations to which students have not yet been exposed, 73% believe that this should be the aim of any future Applied Mathematics syllabus.

3.3 Conceptualisation of mathematics teaching and learning

When asked to rank student activities in order of importance there is support for a pedagogical approach consistent with that promoted by the *Project Maths* initiative. The top four ranked student activities being:

- discussing solution strategies to problems
- justifying solutions to problems
- solving real life problems
- solving open ended problems.

When asked to reflect on their own classroom practice only 18% of respondents agreed that good comprehensive notes are the most important teaching tool that students need in order to be successful in applied mathematics.

Respondents view past examination questions as an important teaching tool. While there was a strong neutral response (25%) in relation to the statement *it is not necessary for students to practice examination questions repeatedly in order to succeed in Applied Mathematics*, only 20% of respondents agreed with that statement.

3.4 Possible frameworks for the new specification

In line with the focus group findings, a consensus failed to emerge on which of the possibilities presented is more likely to prepare students for post-school pathways with a significant mathematical component.

An analysis of the open responses threw up a number of concerns and words of caution

- Respondents expressed doubt over the ability of any of the possibilities presented to broaden the appeal

possibility 2 will be chosen as it is closest to the present situation

- Concerns about school's capacity to timetable the course

principals will not be able to cope with this as an option

- Concerns about teacher capacity to deliver a range of options

extra training for teachers will be needed; facilitation of contact between teachers and third level lecturers.

- Respondents expressed concern that the potential to develop problem solving skills will diminish with the inclusion of content

if we put in matrices, vectors and their applications we could have a wider range of contexts but this will leave them time-poor for problem solving

This subject is about problem solving so anything that is not approached in a problem solving manner has no space here.

3.5 Perspectives on assessment

A need for changes in relation to assessment is evident with less than 50% of respondents expressing the view that the current assessment arrangements are sufficient. Suggestions were made in the open responses of possible alternative methods of assessment that would provide students with more varied opportunities to provide evidence of their achievements. These included

- Case studies
- Open book exams
- Project work.

It was noted that any assessment option would need to be both robust and valid.

3.6 Perspectives on technology

There was an overwhelming consensus (86%) that technology has the potential to revolutionise the teaching of Applied Mathematics. The potential role of technology mentioned in the open responses ranged from using You Tube videos and smartboards as teaching aids, empowering students to use computer algebra systems, to collaboration between schools. Once again the open responses yielded a warning about using technology for its own sake.

The responses to the online questionnaire outlined above are consistent with the focus group findings and point to particular areas for consideration by those developing the new specification

- The role of problem solving in the new specification
- The possibilities for assessment
- The potential role for technology within the teaching and learning of the new specification.

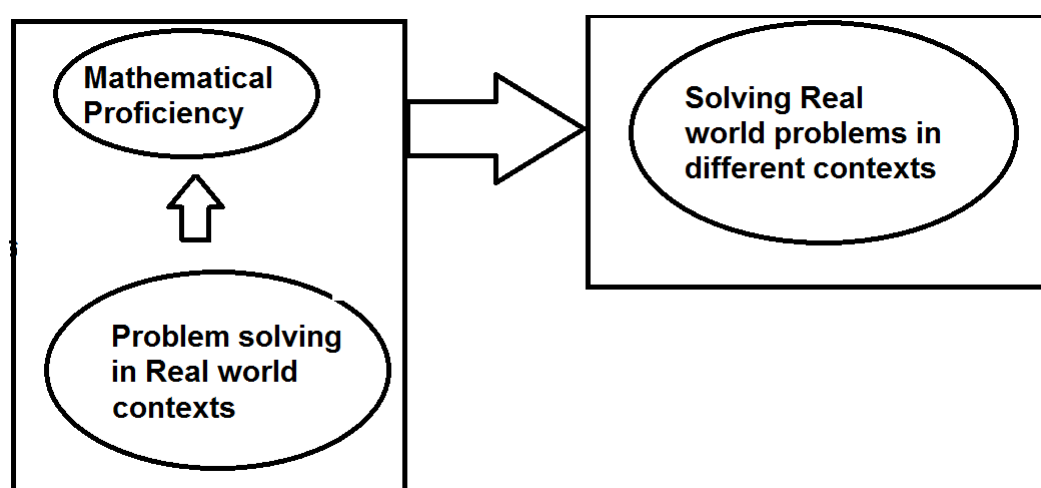
4. Conclusion

When we look beyond the strongly voiced and valid concerns expressed throughout the consultation, there is a commonality between the ideas in the background paper and the motivations and concerns of those who participated in the consultation, and this crystallised around the notion of problem solving.

It was clear from discussions that the mathematical landscape perceived by those consulted is along the lines represented in the diagram below. In their Leaving Certificate Mathematics classes students develop mathematical proficiency. The problems presented are couched in real world contexts to both motivate students and allow them access to the mathematical concepts. Students of the new specification would be enabled, in addition, to apply that mathematical proficiency to solve real world twenty-first century problems set in different contexts.

180 hours *Leaving Certificate Mathematics*

180 hours '*new specification*'



It is recognised that the challenge for the group developing the new specification is deciding on the appropriate contexts and real world twenty-first century problems. Questions they will have to grapple with are

- Should the real world problems be motivated by the mathematics the students are already exposed to?
- Should new mathematics be introduced?
- Should new mathematics be introduced only as part of a major and direct application?

- Should case studies be the vehicle through which students are exposed to real world problems?
- Who will develop the case studies and how will teachers and students access these?
- What is the extent of teacher support and continual professional development?
- In addition, the issues of timetabling, problem solving and content add to the complexity of the agenda set for this review of Applied Mathematics.

In light of the views expressed by those involved in this consultation the brief set out in the *Background Paper for the Review of Applied Mathematics* can be amended as follows:

Brief

The development of the specification will contribute to clarifying the paths that students can take through mathematics at senior cycle, and into related fields of third level education. It will address continuity and progression by categorising possible post-school pathways and considering mathematics curriculum pathways at senior cycle appropriate to each category.

Some consideration will be given to naming the subject/specification and to whether it should be developed at a single level only or at two levels.

In line with current developments, the specification will be student-centred and outcomes-based, and follow the format of other senior cycle specifications:

- Introduction and rationale
- Aim
- Objectives
- Structure
- Key skills
- Learning outcomes
- Assessment
 - Assessment components
 - Assessment criteria.

In general terms, the specification should be aligned with levels 4/5 of the National Framework of Qualifications. If developed as a Higher Level only specification, that alignment should predominantly be towards level 5.

More specifically, the development of the new/revised specification will address

- the structure of the specification and whether it should have a core and options/case studies option and related issues
- whether and how to include any new mathematics content
- how to widen the appeal of the specification by including applications of mathematics to solve real world twenty-first century problems in motivating and exciting contexts
- how its presentation can support teachers in reconceptualising mathematics, teaching and learning
- how to encourage the existing cohort to remain teaching the subject as well as attracting new teachers to the subject
- how student work will be assessed; the provision of multiple, diverse, valid and robust opportunities for students to achieve
- how to embrace technology and collaboration in the learning, teaching and assessment associated with the specification
- the broader context of 'pathways in mathematics' at senior cycle and how this specification relates to those pathways and to existing mathematics specifications.

The work of the Mathematics Development Group will be based, in the first instance, on this brief. In the course of its work and discussions, elaborations of some of these points and additional points may be included in the brief.

Appendix 1

Focus group questions

Q1. What were your initial impressions when you read the background paper?

Q2. The following concerns about the type and quality of Applied Mathematics education that students experience in school are identified in the introduction section 1 (p1,2&3) of the Background Paper

- The narrow appeal of the current syllabus
- The overemphasis on content at the expense of the development of skills and mathematical reasoning
- The reality that students rarely engage with the syllabus for the intended 180 hours
 - The reality that students' experience of learning is often reduced to examination preparation

In your opinion how well has the background paper dealt with these concerns?

What are the challenges?

How might these be dealt with?

Q3. The background paper recognises that there are a number of conflicting views held around the purpose of mathematics education at senior cycle.

Currently mathematics education at senior cycle in Ireland provides education for life.

The background paper draws on international thinking and suggests that a number of pathways should be made available at senior cycle to accommodate the variation in student needs based on their post school aspirations.

In your opinion what are the benefits of this proposed approach

What are the challenges?

Q4. The background paper outlines the variance at an international level in terms of the number of classroom hours available to students to engage with mathematics. For example in Singapore a student who wishes to continue to a third level course with a significant mathematical component and who has a passion and aptitude for mathematics can engage with mathematics for up to 460 hours.

If we take the pathways approach the new specification would be more in the realm of Further Mathematics and therefore would need to expand beyond mathematical physics.

What are benefits of this approach? What are challenges?

Q5. The background paper proposes that an aim of the specification should be to develop problem-solving skills in Mathematics which can be transferred to unique and novel situations to which students have not yet been exposed

What do you see as the advantages of this learning outcome for students?

What teaching and assessment challenges does this give rise to?

Q6. If the purpose of the specification is to broaden the appeal of applied mathematics and prepare students for post-school pathways in your opinion, how likely would each of the suggested possibilities 1 and 2 (pages 18 and 19 of the background paper), be to broaden the appeal?

Why? (for each)

What supports would be needed if these options were provided (for each)? Why?

Q7. *A significant challenge for the development group working on the specification will be to present the material from the core in a way that will motivate and engage learners with a range of interests. Another possibility is to abandon the core and options structure altogether and simply provide curriculum pathways representing a number of specialised areas where mathematics is applied. Thus providing a structure for students to combine their interest in mathematics and mathematical reasoning, with an interest in a specific field of intellectual activity*

- How likely is possibility 3 to overcome these challenges?
- What supports would be needed for option 3 to be successful?

Q8. *Since expectations of what learners should achieve have changed since the Applied Mathematics syllabus was developed over 40 years ago there is a need for reflection on how best to design assessment approaches for the new specification in ways that will give students fairer and more varied opportunities to provide evidence of their achievements, and that will go some way towards measuring the broader competencies set out in the Key Skills Framework (NCCA 2009).*

- What are your views on assessing Applied Mathematics in the twenty first century?
- How might any of the challenges that you have identified be overcome?
- What supports will teachers need for these solutions to be successful?

Q9. *Technology is contributing to a reconceptualising of learning, to expand beyond linear, text-based learning and to engage students who learn best in other ways. Its role in schools has evolved from a contained 'computer class' into a versatile learning tool with the potential to change how concepts are demonstrated, projects assigned and progress assessed*

- How can technology be used to support the teaching and assessment of the new specification?
- How could technology assist in supporting schools in facilitating a range of students' interests? What supports would teachers need?

Appendix 2

The following written submissions were received

- Irish Applied Mathematics Teachers Association (IAMTA)
- Irish Mathematics Teachers Association (IMTA)
- School of Mathematical Sciences UCD
- Personal
 - Eugene Gath UL
 - Dominic Donnelly Applied Mathematics teacher
 - Jim Grannell UCC
 - Sean Connolly

Appendix 3 - The consultation survey

Introduction

The aim of this consultation is to hear the open and honest views of teachers/parents/students and interested parties on the 'Background Paper and Brief for the Review of Applied Mathematics'. The generation of the background paper and brief is the first stage of the curriculum and assessment development process. It provides a background to the development of a specification to replace 'Leaving Certificate Applied Mathematics' which will be published in September 2015. This consultation will involve gathering feedback through this survey, direct submissions and targeted meetings. The consultation process will close on 19 December, 2014. The NCCA would greatly appreciate your feedback. This feedback will inform the work of the mathematics development group. Please read the paper which can be [downloaded here](#). Then complete this questionnaire as fully as possible but feel free to skip any item that is not relevant to you.

Participant's details

*1. I am responding as a ...

- | | |
|--|---|
| <input type="radio"/> Applied Mathematics teacher | <input type="radio"/> Third level student |
| <input type="radio"/> Mathematics Teacher | <input type="radio"/> Parent/ Guardian |
| <input type="radio"/> Non-mathematics Teacher (Post-primary) | <input type="radio"/> Teacher educator |
| <input type="radio"/> Principal/ Deputy Principal (Post-primary) | <input type="radio"/> Pre-service Teacher |
| <input type="radio"/> Principal/ Deputy Principal (Primary) | <input type="radio"/> Third level lecturer/researcher |
| <input type="radio"/> Second level student | <input type="radio"/> Other |

Other (please specify)

2. If you are responding on behalf of a group/organisation, please indicate below whether the submission may be made publicly available and/or whether the group/organisation may be named in the consultation report as a contributor.

	Yes	No
My group/organisation agrees to this submission being made available.	<input type="radio"/>	<input type="radio"/>
My group/organisation agrees to be named as a contributor to the consultation.	<input type="radio"/>	<input type="radio"/>

Please provide the name of the group/organisation on whose behalf you are responding

3. School type (if applicable)

- | | |
|--|---|
| <input type="radio"/> Voluntary secondary school | <input type="radio"/> Vocational school |
| <input type="radio"/> Community school | <input type="radio"/> Community college |
| <input type="radio"/> Comprehensive school | <input type="radio"/> Other |

Other (please specify)

4. You can provide your name and email address to enable us to follow-up on particular issues that you may identify. However, your survey data will be anonymised and we will ensure that no views which you articulate will be attributed to you or be reported in any way that would allow you to be identified.

Name

Email address

Concerns regarding applied mathematics education

The following concerns about the type and quality of applied mathematics education that students experience in school are identified in the introduction section 1(p1,2&3) of the Background Paper

- the narrow appeal of the current syllabus
- the overemphasis on content at the expense of the development of skills and mathematical reasoning.
- the reality that students rarely engage with the syllabus for the intended 180 hours.
- the reality that students' experience of learning is often reduced to examination preparation.

5. Please indicate your level of agreement with each of the following statements.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
An applied mathematics syllabus should be attractive to a small cohort of high achievers.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Applied mathematics students should be exposed to a broader course.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Broadening the current applied mathematics syllabus would increase its appeal.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
An important role of applied mathematics is to provide students with an opportunity to develop 'reasoning and problem solving skills'.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
The current applied mathematics syllabus prepares students to solve particular 'problem types' in mathematical physics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
The relevance of applied mathematics would be enhanced by a greater re-balancing away from content towards reasoning and skills.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Teaching of the applied mathematics syllabus does not end before the mocks to focus on examination preparation.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

If you wish, please outline any additional concerns you may have regarding applied mathematics education.

Purpose(s) of applied mathematics education

The background paper outlines the importance of STEM education at many different levels (page 5).

6. Please indicate which of the following statements you most agree with

- ☐ Mathematics education at senior cycle should prepare students to participate fully in modern society.
- ☐ Mathematics education at senior cycle should focus on preparing students for further study in mathematics related disciplines.
- ☐ Mathematics education at senior cycle should prepare students to participate fully in modern society and at the same time prepare them for further study in mathematics related disciplines.

Please explain your answer

Engaging students with senior cycle mathematics

The tables on pages 9-11 of the background paper show variance at an international level in terms of the number of classroom hours available to students to engage with mathematics. For example, in Singapore a student who wishes to continue to a third level course with a significant mathematical component and who has a passion and aptitude for mathematics can engage with mathematics for up to 460 hours.

7. Which of these statements best reflects your opinion?

- ☐ Students who wish to pursue further study in mathematics related disciplines should have the opportunity in school to engage with mathematics for longer than the 180 hours allocated to the mathematics specification.
- ☐ 180 hours of mathematics in senior cycle is sufficient for all students regardless of their future aspirations.

Please explain your answer

8. Page 12 of the background paper gives an overview of potential pathways for mathematics education in Ireland. Regarding this overview please indicate your level of agreement with the following statements.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
Broadening the specification will contribute to enhancing the appeal of applied mathematics	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
The broader specification taken in conjunction with HL mathematics represents a suitable pathway for students who wish to proceed to courses in 3rd level with a significant mathematical component.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Readiness for post-school pathways

The background paper pages 13-14, discusses the 'habits of mind' students develop in post-primary school and bring with them to post-school courses. Towards Learning (NCCA 2009) sets out five key skills essential to learning at senior cycle.

1. Critical and creative thinking
2. Communicating
3. Information processing
4. Being personally effective
5. Working with others

Q 9-11 seek your opinion on these key skills and habits of mind.

9. Page 3 of the background paper states....'Leaving Certificate Applied Mathematics is also promoted as a subject that enhances students' problem solving skills. However, with its emphasis on content as opposed to the development of skills and mathematical reasoning students are not problem solving per se but rather, learning to solve particular problem types in mathematical physics.'

Please select the statement which best represents your view.

- ☐ The current applied mathematics syllabus develops problem solving skills in the particular types of problems that students have been taught.
- ☐ The current applied mathematics syllabus develops problem solving skills which can be transferred within problem types to which students have been exposed.
- ☐ The current applied mathematics syllabus develops problem solving skills which can be transferred to unique and novel situations to which students have not yet been exposed.

Please explain your answer

10. Please select the statement which best represents your view.

- ☐ The aim of any future applied mathematics syllabus should be to develop problem solving skills in the particular types of problems that students have been taught.
- ☐ The aim of any future applied mathematics syllabus should be to develop problem solving skills which can be transferred within problem types to which students have been exposed.
- ☐ The aim of any future applied mathematics syllabus should be to develop problem solving skills which can be transferred to unique and novel situations to which students have not yet been exposed.

Please explain your answer

11. Please indicate your level of agreement with the following statements

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
Specific content knowledge is more important than critical thinking, analytic thinking and problem solving.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
A revised applied mathematics specification should provide students with an opportunity to consolidate their learning of LC Mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Conceptualisation of mathematics, teaching and learning

Reflecting on practice in the mathematics/applied mathematics classroom.

12. Rank the following student activities 1-13 in order of importance (1 = most important; 13 = least important).

Watching the teacher solve problems and copying solutions from the board.

Practicing problems from text books

Justifying solutions to problems

Solving real life problems

Practicing past examination questions

Solving open ended problems

Discussing solution strategies to problems

Presenting work to their peers

Using notes to solve problems

Struggling with a problem

Working alone

Working collaboratively with peers

Using technology

13. Please include other comments you wish to make on the importance of any of the teaching methods from Q12 above.

14. Again, reflecting on your own practice or opinions please indicate your level of agreement with the following statements about classroom pedagogies.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
Good comprehensive notes are the most important teaching tool that students need in order to succeed in applied mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
It is not necessary for students to practice exam questions repeatedly in order to succeed in applied mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Discussing solution strategies to problems with their peers is not necessary for students to succeed in applied mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Students justifying their solution strategies with their peers is important to facilitate success in applied mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Productive struggle, i.e. trying a question and failing, is important for students to be successful in applied mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
It should be possible to complete the applied mathematics syllabus before the mocks begin in order to leave plenty of time for revision.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

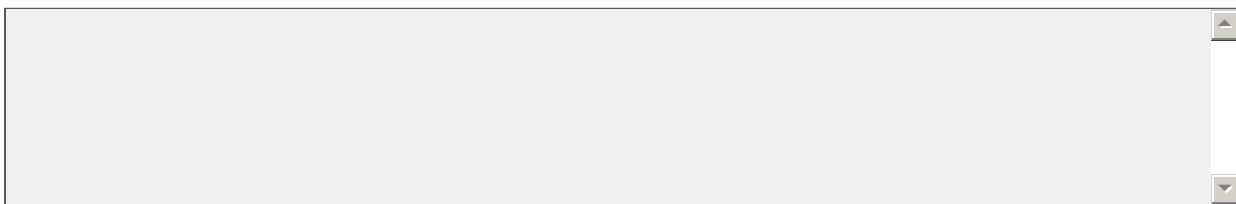
Post-school pathways

If the purpose of the specification is to broaden the appeal of applied mathematics and prepare students for post-school pathways, please refer to the three possibilities outlined on pages 18 and 19 of the background paper.

15. Which of the three possibilities is most likely to broaden the appeal?

- ☐ Possibility 1. All students study the core and select one option from a comprehensive list. (page 18)
- ☐ Possibility 2. The weighting of the core is reduced; all students study the core and select two options from a comprehensive list.(page 19)
- ☐ Possibility 3. Students engage with the material in the context of a chosen area of application.(page 19)

Please explain why you think this possibility will broaden the specification.



16. Which of the three possibilities is most likely to prepare students for post-school pathways with a significant mathematical component?

- ☐ Possibility 1. All students study the core and select one option from a comprehensive list. (page 18)
- ☐ Possibility 2. The weighting of the core is reduced; all students study the core and select two options from a comprehensive list.(page 19)
- ☐ Possibility 3. Students engage with the material in the context of a chosen area of application.(page 19)

Please explain why you think this possibility is most likely to prepare students for post-school pathways with a significant mathematical component.



Perspectives on assessment

The background paper offers a perspective on Assessment (pages 21-22). Since expectations of what learners should achieve have changed since the Applied Mathematics syllabus was developed over 40 years ago, there is a need for reflection on how best to design assessment approaches for the new specification in ways that will give students fairer and more varied opportunities to provide evidence of their achievements, and that will go some way towards measuring the broader competencies set out in the Key Skills Framework (NCCA 2009).

Regarding this please respond to question 17 below.

17. Please indicate your level of agreement with the following statements.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
The current assessment arrangement is sufficient to assess student achievement in applied mathematics.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Students should be given more varied opportunities to provide evidence of their achievements.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

You may wish to add further comments relating to the assessment approaches documented in appendix 3 of the background paper. If so, please use the space below.

Perspectives on the role of technology.

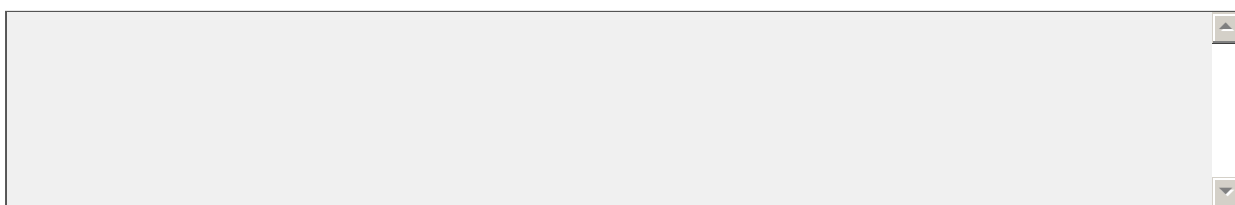
The background paper offers a perspective on the role of technology in the new specification (pages 22-23). Technology is contributing to a re-conceptualising of learning, to expand beyond linear, text-based learning and to engage students who learn best in other ways. Its role in schools has evolved from a contained 'computer class' into a versatile learning tool with the potential to change how concepts are demonstrated, projects assigned and progress assessed.

Regarding this please answer question 18 below.

18. Which of these statements best reflects your opinion.

- ☐ Technology has the potential to revolutionise the teaching of applied mathematics.
- ☐ There is no role for technology in the teaching of applied mathematics.

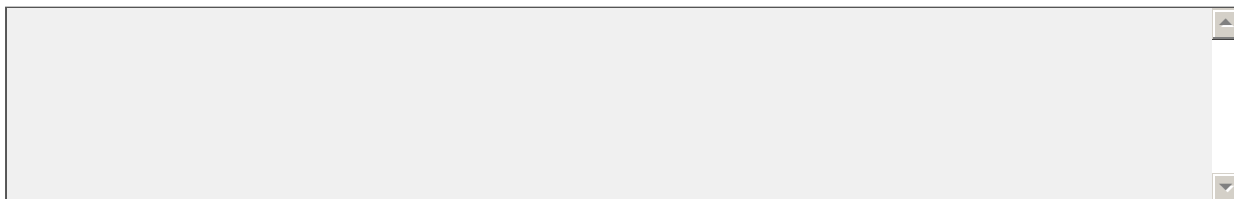
Please explain your answer.



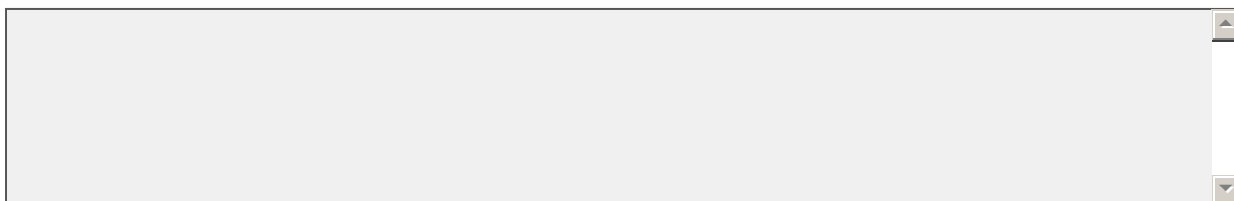
Perspective on supports

The next two questions ask for your opinion on the supports that need to be put in place to facilitate the review of applied mathematics.

19. If the applied mathematics syllabus was to be broadened what type of supports would need to be put in place?

A large, empty rectangular text input area with a light gray background. On the right side, there is a vertical scrollbar with a small upward-pointing arrow at the top and a downward-pointing arrow at the bottom.

20. If the specification is to be part of a pathway through mathematics education for those wishing to pursue mathematically related courses in 3rd level what supports would need to be put in place?

A large, empty rectangular text input area with a light gray background. On the right side, there is a vertical scrollbar with a small upward-pointing arrow at the top and a downward-pointing arrow at the bottom.

The brief for the review of applied mathematics

21. Is there any final comment you would like to make on the brief for the review of applied mathematics? If so, please use the space below.



Thank you

Thank you for taking the time to complete this questionnaire. Please do not forget to press 'Submit' when you are finished.

If you have completed a hard copy of this questionnaire, please return it to:

Applied Mathematics Background Paper Consultation,
NCCA,
James Fintan Lalor Ave,
Portlaoise,
Co. Laois.

A summary of the findings will be published on the NCCA website at the end of the consultation.