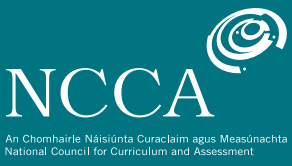
Mathematics in Early Childhood and Primary Education (3-8 years)

Executive Summaries

NCCA

An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment



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Preface

The mathematical literacy of our children and young people is key to their participation in learning and education, and to their future life chances and employment opportunities. Increasingly, a high standard of mathematical skills generally is an important element in Ireland's economic development. *Literacy and numeracy for learning and life,* Ireland's national strategy to improve literacy and numeracy among children and young people, acknowledges the importance of mathematics and presents a shared goal for numeracy for parents and communities; practitioners and teachers; and leadership in schools.

The Project Maths initiative, which began in post-primary schools in 2008, emphasises the development of conceptual understanding, reasoning and problem solving skills. Since the development of mathematical concepts begins very early in a child's education it makes sense that we turn our attention now to *what* a child learns in mathematics and *how*, beginning with the early years of primary school. *Aistear: the Early Childhood Curriculum Framework* (2009) also highlights the potential and promise of a more child-centred approach to the development of children's early mathematical literacy.

This booklet contains Executive Summaries of two research reports which the NCCA commissioned to support the development of the Primary Mathematics Curriculum:

- *Mathematics in Early Childhood and Primary Education (3-8 years), Definitions, Theories, Development and Progression*
- *Mathematics in Early Childhood and Primary Education (3-8 years), Teaching and Learning.*

The contents of the full reports, which are available at *ncca.ie/primarymaths*, serve to enliven and enlighten our understanding and discussion of children's mathematical learning and development in the early childhood and primary years, and the kinds of curriculum and assessment supports needed. In order to

broaden access to key messages from the reports, the authors have also prepared a series of short podcasts (available at *ncca.ie/primarymaths*) in which they discuss important ideas in the reports for parents, practitioners and teachers. The authors are to be commended on these excellent reports which deepen and enrich the context for work on the Primary Mathematics Curriculum.

The NCCA is committed to quality in developing curriculum and assessment which is both evidence-based and informed by practice. These research reports mark the beginning of Council's work to develop the new mathematics curriculum for primary schools. We look forward to a wide-ranging engagement with all concerned in this important task.

Brigid McManus
Chairperson, NCCA

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The executive summaries of reports No. 17 and No. 18 are available online at *ncca.ie/primarymaths*. The online versions include some hyperlinks which appear as text on dotted lines in this print copy.

RESEARCH REPORT NO. 17

Definitions, Theories, Development and Progression

The review of research on mathematics learning of children aged 3–8 years is presented in two reports. These are part of the NCCA's Research Report Series (ISSN 1649–3362). The first report (Research Report No. 17) focuses on theoretical aspects underpinning the development of mathematics education for young children. The second report (Research Report No. 18) is concerned with related pedagogical implications. The key messages from Report No. 17 are presented in this Executive Summary.

A View of Mathematics

Both reports are underpinned by a view of mathematics espoused by Hersh (1997). That is, mathematics as 'a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context' (p. xi). Mathematics is viewed not only as useful and as a way of thinking, seeing and organising the world, but also as aesthetic and worthy of pursuit in its own right (Zevenbergen, Dole, & Wright, 2004). All children are viewed as having an ability to solve mathematical problems, make sense of the world using mathematics, and communicate their mathematical thinking.

Context

The context in which this report is presented is one in which there is a growing awareness of the importance of mathematics in the lives of individuals, in the economy and in society more generally. In parallel with this there is a growing realisation of the importance of the early childhood years as a time when

children engage with many aspects of mathematics, both at home and in educational settings (Ginsburg & Seo, 1999; Perry & Dockett, 2008). Provision for early childhood education in Ireland has also increased. A recent development is free preschool education for all children in the year prior to school entry. In addition, a new curriculum framework, *Aistear* (National Council for Curriculum and Assessment [NCCA], 2009a; 2009b), is available to support adults in developing children's learning from birth to six years. At the same time, however, there are concerns about the levels of mathematical reasoning and problem-solving amongst school-going children, as evidenced in recent national and international assessments and evaluations at primary and post-primary levels (e.g., Eivers et al., 2010; Perkins, Cosgrove, Moran & Shiel, 2012; Jeffes et al., 2012). While the 1999 Primary School Mathematics Curriculum (PSMC) has been well received by teachers (NCCA, 2005), the Inspectorate of the then Department of Education and Science identified some difficulties with specific aspects of implementation (DES, 2005). The current report envisions a revised PSMC that is responsive to these concerns, that recognises the importance of building on children's early engagement with mathematics, and which takes account of the changing demographic profile of many educational settings, and the increased diversity among young children.

Definitions of Mathematics Education

Current views of mathematics education are inextricably linked with ideas about equity and access and with the vision that mathematics is for all (Bishop & Forganz, 2007), i.e. all children should have opportunities to engage with and benefit from mathematics education and no child should be excluded.

Mathematics education is seen as comprising a number of mathematical practices that are negotiated by the learner and teacher within broader social, political and cultural contexts (Valero, 2009). An interpretation of mathematics that includes numeracy but is broader should underpin efforts towards curricular reform in

Ireland. This report identifies mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition) (NRC, 2001) as a key aim of mathematics education. It is promoted through engagement with processes such as connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising. All of these are encompassed in the overarching concept of mathematization. This involves children interpreting and expressing their everyday experiences in mathematical form and analysing real world problems in a mathematical way through engaging in these key processes (Ginsburg, 2009a; Treffers & Beishuizen, 1999). Thus mathematization is identified as a key focus of mathematics education and as such it is given considerable attention in this report. Mathematics education should address the range of mathematical ideas that all children need to engage with. It should not be limited to number.

Theoretical Perspectives

Cognitive and sociocultural perspectives provide different lenses with which to view mathematics learning and the pedagogy that can support it (Cobb, 2007). Cognitive perspectives are helpful in focusing on individual learners while sociocultural perspectives are appropriate when focusing on, for example, pedagogy (Cobb & Yackel, 1996). Sociocultural, cognitive perspectives and constructionism all offer insights which can enrich our understanding of issues related to the revision of the curriculum. They do so by providing key pointers to each of the elements of learning, teaching, curriculum and assessment. Used together they can help in envisaging a new iteration of the PSMC.

In this report, learning mathematics is presented as an active process which involves meaning making, the development of understanding, the ability to participate in increasingly skilled ways in mathematically-related activities and the development of a mathematical identity (Von Glasersfeld, 1984; Rogoff, 1998; Lave & Wenger, 1991). Learning also involves the effective use of key

tools such as language, symbols, materials and images. It is seen to be supported by participation in the community of learners engaged in mathematization, in small-group and whole class conversations. The proactive role of the teacher must be seen to involve the creation of a zone of proximal development, the provision of scaffolding for learning and the co-construction of meaning with the child based on awareness and understanding of the child's perspective (e.g., Bruner, 1996). It also involves a dialogical pedagogy of argumentation and discussion designed to support effective conceptual learning and the ability for teachers to act contingently (e.g., Corcoran, 2012).

Language and Communication

Cognitive/constructivist and sociocultural perspectives on learning emphasise the key role of language in supporting young children's mathematical development. Emerging learning theories point to the importance of mathematical discourse as a tool to learn mathematics (e.g., Sfard, 2007). In addition to introducing young children to mathematical vocabulary, it is important to engage them in 'math talk' – conversations about their mathematical thinking and reasoning (Hufferd-Ackles, Fuson & Sherin, 2004). Such talk should occur across a broad range of contexts, including unplanned and planned mathematics activities and activities such as storytelling or shared reading, where mathematics may be secondary. Children at risk of mathematical difficulties, including those living in disadvantaged circumstances, may need additional, intensive support to develop language and the ability to participate in mathematical discourse (Neuman, Newman & Dwyer, 2011).

Research indicates an association between the quality and frequency of mathematical language used by carers, parents and teachers as they interact with young children, and children's development in important aspects of mathematics (Klibanoff et al., 2006; Gentner, 2003; Levine et al., 2012). This highlights the importance of adults modelling mathematical language and encouraging young children to use such language. Conversations amongst

children about mathematical ideas are also important for mathematical development (e.g., NRC, 2009).

Defining Goals

The goal statements of a curriculum should be aligned with its underlying theory. Curriculum goals should reflect new emphases on ways to develop children's mathematical understandings and to foster their identities as mathematicians (Perry & Dockett, 2002; 2008). This report proposes that processes and content should be clearly articulated as related goals (e.g., mathematization can be regarded as both a process and as content since as children engage in processes e.g., connecting, they construct new and/or deeper understandings of content). This contrasts with the design of the Primary School Mathematics Curriculum (PSMC), where content and processes are presented separately, and content is emphasised over processes. An approach in which processes are foregrounded, but content areas are also specified, is consistent with a participatory approach to mathematics learning and development.

General goals need to be broken down for planning, teaching and assessment purposes. This can be done through identifying critical ideas i.e., the shifts in mathematical reasoning required for the development of mathematical concepts (e.g., Simon, 2006; Sarama & Clements, 2009). An understanding of this framework enables teachers to provide support for children's progression towards curriculum goals.

The Development of Children's Mathematical Thinking

The idea of stages of development in children's mathematical learning (most often associated with Piaget) has now been replaced with ideas about developmental/learning paths. This is a relatively recent area of research in mathematics education

(Daro et al., 2011) and as such is still under development. Learning paths are also referred to as learning trajectories. They indicate the sequences that apply in a general sense to development in the various domains of mathematics (e.g., Fosnot & Dolk, 2001; Sarama & Clements, 2009; van den Heuvel-Panhuizen, 2008). This report envisages that general learning paths will provide teachers with a basis for assessing and interpreting the mathematical development in their own classroom contexts, and will lead to learning experiences matched to individual children's needs.

There is variation in the explication of learning paths, for example, linear/nonlinear presentation, level of detail specified, mapping of paths to age/grade, and role of teaching. Different presentations reflect different theoretical perspectives. An approach to the specification of learning paths that is consistent with sociocultural perspectives is one which recognises the paths as

i. provisional, as many children develop concepts along different paths and there can never be certainty about the exact learning path that individual children will follow as they develop concepts

ii. not linked to age, since this suggests a normative view of mathematics learning

iii. emerging from engagement in mathematical-rich activity with children reasoning in, and contributing to, the learning/teaching situation (e.g., Fosnot & Dolk, 2001; Stigler & Thompson, 2012; Wager & Carpenter, 2012).

Assessing and Planning for Progression

Of the assessment approaches available, formative assessment offers most promise for generating a rich picture of young children's mathematical learning (e.g., NCCA, 2009b; Carr & Lee, 2012). Strong conceptual frameworks are important for supporting teachers' formative assessments (Carr & Lee, 2012; Ginsburg, 2009a; Sarama & Clements, 2009). These influence what teachers recognise as significant learning, what they take note of and what aspects of

children's activity they give feedback on. There is a range of methods (observation, tasks, interviews, conversations, pedagogical documentation) that can be used by educators to assess and document children's mathematics learning and their growing identities as mathematicians. Digital technologies offer particular potential in this regard. These methods are challenging to implement and require teachers to adopt particular, and for some, new, perspectives on mathematics, mathematics learning and assessment. Constructing assessments which enlist children's agency (for example, selecting pieces for inclusion in a portfolio or choosing particular digital images to tell a learning story) has many benefits. One benefit is the potential for the inclusion of children's perspectives on their learning (Perry & Dockett, 2008).

In the main, the current literature affords scant support for the use of standardised tests with children in the age range 3–8 years (e.g., Mueller, 2011). More structured teacher-initiated approaches and the use of assessment within a diagnostic framework may be required on some occasions, for example, when children are at risk of mathematical difficulties. However, research indicates a range of factors problematising the use of standardised measures with young children (e.g., Snow & Van Hemel, 2008).

The complex variety of language backgrounds of a significant minority of young children presents a challenge in the learning, teaching and assessment of mathematics. Children for whom the language of the home is different to that of the school need particular support. That support should focus on developing language, both general and mathematical, to maximise their opportunities for mathematical development and their meaningful participation in assessment (Tabors, 2008; Wood & Coltman, 1998). Educators carrying out assessment procedures such as interviews, observations or tasks in an immersion context have the dual purpose of assessing and evaluating both the mathematical competences and language competences of the child, to gain a full picture. Dual language assessment is particularly desirable in this context (Murphy & Travers, 2012; Rogers, Lin & Rinaldi, 2011).

Addressing Diversity

Mathematics 'for all' implies a pedagogy that is culturally sensitive and takes account of individuals' ways of interpreting and making sense of mathematics (Malloy, 1999; Fiore, 2012). An issue of concern is the limitations of norms-based testing which can disadvantage certain groups. This indicates the need to use a diverse range of assessment procedures to identify those who are experiencing learning difficulties in mathematics.

The groups of individuals that often require particular attention in the teaching and learning of mathematics are 'exceptional' children (those with developmental disabilities or who are especially talented at mathematics) (Kirk, Gallagher, Coleman, & Anastasiow, 2012). These individuals do not require distinctive teaching approaches, but there is a need to address their individual needs. In particular, the use of multi-tiered tasks in which different levels of challenge are incorporated is advocated (Fiore, 2012).

In addition, this report identifies the need to provide parents and educators with particular supports to ensure a mathematically-interactive and rich environment for children aged 3–8 years. It also indicates that the intensity of the support needs to vary according to the needs of particular groups of children (e.g., Ehrlich, Levine, & Goldin-Meadow, 2006).

Key Implications

The following are the key implications that arise from this report for the development of the mathematics curriculum for children aged 3–8 years:

- In the curriculum, a view of all children as having the capacity to engage with deep and challenging mathematical ideas and processes from birth should be presented. From this perspective, and in order to address on-going concerns about mathematics at school level, a curriculum for 3–8 year-old

children is critical. This curriculum needs to take account of the different educational settings that children experience during these years.

- The curriculum should be developed on the basis of conversations amongst all educators, including those involved in the NCCA's consultative structures and processes, about the nature of mathematics and what it means for young children to engage in doing mathematics. These conversations should be informed by current research, as synthesised in this report and in Report No. 18, which presents a view of mathematics as a human activity that develops in response to everyday problems.

- The overall aim of the curriculum should be the development of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition). As mathematization plays a central role in developing proficiency, the processes of mathematization should permeate all learning and teaching activities. These include connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising. (*Chapter 1*)

- The curriculum should foreground mathematics learning and development as being dependent on children's active participation in social and cultural experiences, while also recognising the role of internal processes. This perspective on learning provides a powerful theoretical framework for mathematics education for young children. Such a framework requires careful explication in the curriculum and its implications for pedagogy should be clearly communicated. (*Chapter 2*)

- In line with the theoretical framework underpinning the curriculum, mathematical discourse (math talk) should be integral to the learning and teaching process. The curriculum should also promote the development of children's mathematical language in learning situations where mathematics development may not be the primary goal. Particular attention should be given to providing intensive language support, including mathematical

language, to children at risk of mathematical difficulties. (*Chapter 3*)

- The goal statements of the curriculum should be aligned with its underlying theory. An approach whereby processes are foregrounded but content areas are also specified is consistent with a participatory approach to mathematics learning and development. In the curriculum, general goals need to be broken down for planning, teaching and assessment purposes. Critical ideas indicating the shifts in mathematical reasoning required for the development of key concepts should be identified. (*Chapter 4*)

- Based on the research which indicates that teachers' understanding of developmental progressions (learning paths) can help them with planning, educators should have access to information on general learning paths for the different domains. Any specification of learning paths should be consistent with sociocultural perspectives, which recognise the paths as provisional, non-linear, not age-related and strongly connected to children's engagement in mathematically-rich activity. Account needs to be taken of this in curriculum materials. Particular attention should be given to the provision of examples of practice, which can facilitate children's progression in mathematical thinking. (*Chapter 5*)

- The curriculum should foreground formative assessment as the main approach for assessing young children's mathematical learning, with particular emphasis on children's exercise of agency and their growing identities as mathematicians. Digital technologies offer particular potential in relation to these aspects of development. The appropriate use of screening/diagnostic tests should be emphasised as should the limitations of the use of standardised tests with young children. The curriculum should recognise the complex variety of language backgrounds of a significant minority of young children and should seek to maximise their meaningful participation in assessment. (*Chapter 6*)

- A key tenet of the curriculum should be the principle of 'mathematics for all'. Central to this is the vision of a multicultural curriculum which values the many ways in which children make sense of mathematics. While there are some groups or individuals who need particular supports in order to enhance their engagement with mathematics, in general distinct curricula should not be advocated. (*Chapter 7*)
- Curriculum developments of the nature described above are strongly contingent on concomitant developments in pre-service and in-service education for educators at preschool and primary levels.

RESEARCH REPORT NO. 18

Teaching and Learning

The review of research on mathematics learning of children aged 3–8 years is presented in two reports. These are part of the NCCA's Research Report Series (ISSN 1649–3362). The first report (Research Report No. 17) focuses on theoretical aspects underpinning the development of mathematics education for young children. The second report (Research Report No. 18) is concerned with related pedagogical implications. The key messages from Report No. 18 are presented in this Executive Summary.

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Both volumes are underpinned by a view of mathematics espoused by Hersh (1997): mathematics as 'a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context' (p. xi). Mathematics is viewed not only as useful and as a way of thinking, seeing and organising the world, but also as aesthetic and worthy of pursuit in its own right (Zevenbergen, Dole, & Wright, 2004). All children are viewed as having an ability to solve mathematical problems, make sense of the world using mathematics, and communicate their mathematical thinking. This shift in perspective demands a change in pedagogy – in particular it puts the teaching-learning relationship at the heart of mathematics.

Context

In Report No. 17 we argue that the overall aim of the curriculum should be the development of mathematical proficiency (conceptual understanding, procedural

fluency, strategic competence, adaptive reasoning, and productive disposition) (National Research Council [NRC], 2001). As mathematization plays a central role in developing proficiency, the processes of mathematization should permeate all learning and teaching activities. These include connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising. Foregrounding mathematical proficiency as the aim of mathematics education has the potential to change the kind of mathematics and mathematical learning that young children experience. As it demands significant changes in pedagogy, curriculum and curricular supports (Anthony & Walshaw, 2007), it also poses challenges that are wide-ranging and systemic.

The development of mathematical proficiency begins in the preschool years, and individuals become increasingly mathematically proficient over their years in educational settings. This implies that educators in the range of early childhood settings need to develop effective pedagogical practices that engage learners in high-quality mathematics experiences. There is a concomitant need to address issues related to curriculum content and presentation. In particular, the questions of how to develop a coherent curriculum and how to formulate progressions in key aspects of mathematics are important. The view of curriculum presented in this report is both wide and dynamic. It is recognised that the mathematics education of young children extends beyond the walls of the classroom: family and the wider community can make a significant contribution to children's mathematical achievement (e.g., Sheldon & Epstein, 2005).

Pedagogy

It is impossible to think about good mathematics pedagogy for children aged 3–8 years without acknowledging that much early mathematical learning occurs in the context of children's play (e.g., Seo & Ginsburg, 2004). Educators need to understand how mathematics learning is promoted by young children's engagement in play, and how best they can support that learning. For instance, adults can help children to maximise their learning by helping them to represent

and reflect on their experiences (e.g., Perry & Dockett, 2007a). Learning through play is seen as fundamental to good mathematics pedagogy in early childhood. It assumes varying degrees of emphasis depending on the age of the child. Recent research points to a number of other important principles which underpin good mathematics pedagogy for children aged 3–8 years (e.g., Anthony & Walshaw, 2009a; NRC, 2005). These principles focus on people and relationships, the learning environment and learners. Features of good mathematics pedagogy can be identified with reference to these principles. Both the principles and the features of pedagogy are consistent with the aim of helping children to develop mathematical proficiency. They pertain to all early educational settings, and are important in promoting continuity in pedagogical approaches across settings.

Practices

Good mathematics pedagogy incorporates a number of meta-practices (i.e., overarching practices) including the promotion of math talk, the development of a productive disposition, an emphasis on mathematical modeling, the use of cognitively challenging tasks, and formative assessment. The literature offers a range of perspectives, and advice, as to the issues for educators in integrating these elements into their practices. In doing so, the vision of 'mathematics for all' is supported.

Good mathematics pedagogy can be enacted when educators engage children in a variety of mathematically-related activities across different areas of learning. The activities should arise from children's interests, questions, concerns and everyday experiences. A deep understanding of the features of good pedagogy should inform the ways in which educators engage children in mathematically-related activities such as play, story/picture-book reading, project work, the arts and physical education. The potential of these activities for developing mathematical proficiency can best be realised when educators focus on children's mathematical sense-making. In addition, educators need to

maximise the opportunities afforded by a range of tools, including digital tools, to mediate learning.

Curriculum Development

Goals, coherent with the aim of mathematical proficiency, should be identified. These goals relate both to process and content. The processes of mathematization, that is, communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, should be foregrounded. In line with the principle of 'mathematics for all', each of the five content domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance – should be given appropriate attention.

Goals need to be broken down for planning, teaching and assessment purposes. Learning paths can be helpful for this purpose. As is outlined in Report No. 17, differences in the ways learning paths are presented in the literature rest largely on their theoretical underpinnings. For example, developmental progressions described by Sarama and Clements (2009) are finely grained and age-related, whereas the TAL[1] trajectories developed in the context of Realistic Mathematics Education (van den Heuvel-Panhuizen, 2008) are characterised by fluidity and the role of context. In line with a sociocultural approach to the learning of mathematics, we advocate that learning paths be used in a flexible way to posit shifts in mathematical reasoning, i.e. critical ideas in each of the domains. Narrative descriptors of critical ideas can be used to inform planning and assessment. Learning outcomes, relating to content domains and processes, can then be derived from a consideration of the goals, learning paths and narrative descriptors. The figure below shows an emerging curriculum model highlighting how the relationships between the different elements may be conceptualised.

1 In Dutch, learning-teaching trajectories are referred to as TALs (i.e., Tussendoelen Annex Leerlijinen).

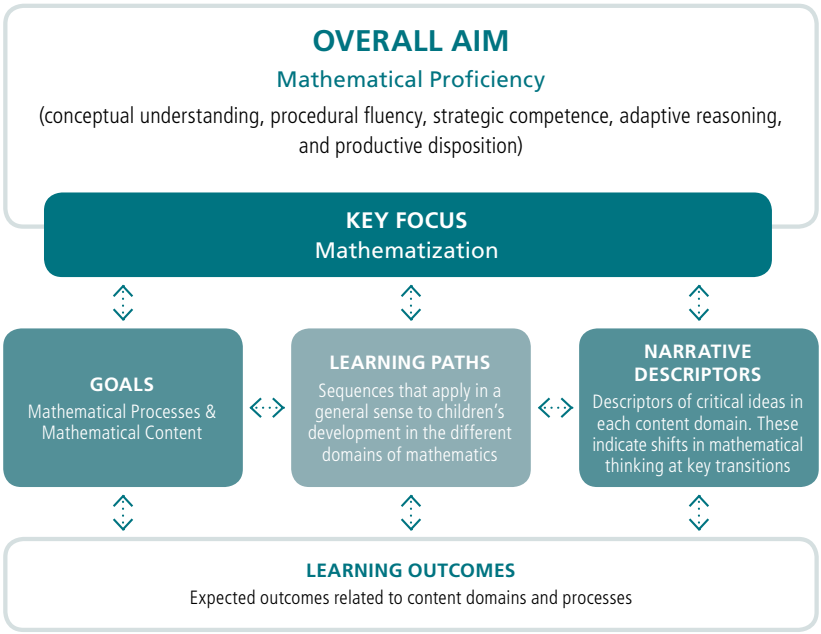


Figure ES.1: Emerging Curriculum Model

Curricular Issues

While the specification of processes and content in the mathematics curriculum is critically important, attention should also be given to issues that relate to curriculum access and curriculum implementation. This is based on the premise that the curriculum must serve all children, including exceptional children (those with developmental delays and those with exceptional talent) and children in culturally diverse contexts. Other key issues include the timing of early

intervention, the allocation of time to mathematics in early learning settings, and how best to achieve the integration of mathematics across the curriculum.

Consistent with Lewis and Norwich's (2005) concept of continua of common teaching approaches that can be subject to varying degrees of intensity depending on children's needs, modifications to the mathematics curriculum for children with special education needs are proposed. Mathematically-talented children should be supported in deepening their understanding of and engagement with the existing curriculum rather than being provided with an alternative one. In the case of English-language learners, and children attending Irish-medium schools, the key role of mathematical discourse and associated strategies in enabling access to the language in which the curriculum is taught are emphasised (e.g., Chapin, O'Connor, & Anderson, 2009). Attention to language is also highlighted as a critical issue in raising the mathematics achievement of children in DEIS schools. More generally, it is noted that there is now strong research indicating that additional support should be provided at an earlier stage than is indicated in current policy documents (e.g. Dowker, 2004; 2009). There is a need to allocate sustained time to mathematics to ensure that all children engage in mathematization. Dedicated and integrated time provision is recommended. The value of integrating mathematics across areas of learning is recognised, though it is acknowledged that relatively little research is available on how best to achieve this.

Partnership with Parents

In line with the emphasis on parental involvement in the *National Strategy to Improve Literacy and Numeracy among Children and Young People (2011–2020)* (Department of Education and Skills [DES], 2011a), the key role of parents in supporting children to engage in mathematics is emphasised. There is a range of activities in which parents can engage with schools so that both parents and educators better understand children's mathematics learning. However, it is

acknowledged that research on parental involvement in mathematics lags behind similar research relating to parental involvement in reading literacy.

In the literature on parental involvement, the need to establish a continuous, two-way flow of information about children's mathematics learning between educators and parents is a key theme. There is potential for technology to support this. Strategies designed to support parents to better understand their child's mathematical learning include observation of and discussion on children's engagement in mathematical activities in education settings. Mechanisms are required to inform parents about the importance of mathematics learning in the early years, and what constitutes mathematical activity and learning for young children. The significant role that parents play in the mathematical development of their children should be foregrounded.

Teacher Preparation and Development

Curriculum redevelopment is strongly contingent on parallel developments in pre-service and in-service education for educators across the range of settings. In particular, professional development programmes need to focus on the features of good mathematics pedagogy and the important meta-practices that arise from these.

In order for teachers to foster mathematical proficiency in children, they themselves need to be mathematically proficient. Therefore, teacher preparation courses need to provide opportunities for pre-service teachers to engage in rich mathematical tasks. Educators need to develop mathematical knowledge for teaching through a collaborative focus on teaching and learning of mathematics. They need opportunities to notice children's engagement in mathematics and responses to mathematical ideas. Case studies of practice are valuable tools in this regard. These can be used by pre-service (and in-service) teachers to question and critique the practice of others in order to develop 'local knowledge of practice' (Cochran-Smith, 2012, p. 46).

Among the recommendations for the continuing professional development of teachers (CPD) is investment in stronger systems of clinical supervision across the preparation-induction boundary (Grossman, 2010). The notion of clinical supervision could mean an emphasis on developing good mathematics teaching practices through collaborative review and reflection on existing practice. This is important because inquiry as a stance has been advocated as a successful key to teacher change (Jaworski, 2006). In this regard, lesson study is a practice that is currently foregrounded in the literature as a significant development in school-based professional development (e.g., Corcoran & Pepperell, 2011; Fernández, 2005). In lesson study, publicly available records of practice or 'actionable artifacts' are important by-products (Lewis, Perry, & Murata, 2006, p. 6). The practice offers opportunities at school and classroom level for enactment of critical inquiry into mathematics lessons.

Key Implications

The key implications for the redevelopment of the mathematics curriculum arising from the review of research presented in this report are as follows:

- The curriculum should be coherent in terms of aims, goals relating to both processes and content, and pedagogy. (*Chapter 1*, *Chapter 3*)
- The processes of mathematization, that is, communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, should be foregrounded in curriculum documentation and should be central to the mathematical experiences of all children. (*Chapter 2*, *Chapter 3*)
- The redeveloped mathematics curriculum needs to acknowledge and build on the pedagogical emphases in *Aistear*. (*Chapter 2*)
- In order to facilitate transitions, educators across early education settings need to communicate about children's mathematical experiences and the

features of pedagogy that support children's learning. (*Chapter 1*)

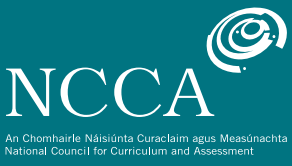
- The principles and features of good mathematics pedagogy as they pertain to people and relationships, the learning environment, and the learner, should be emphasised. (*Chapter 1*)
- The overarching meta-practices and the ways in which they permeate learning activities should be clearly explicated. (*Chapter 2*)
- Educators should be supported in the design and development of rich and challenging mathematical tasks that are appropriate to their children's learning needs. (*Chapter 2*, *Chapter 5*)
- The curriculum should exemplify how tools, including digital tools, can enhance mathematics learning. (*Chapter 2*)
- Children should engage with all five content domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance. The strand of Early Mathematical Activities as presented in the current PSMC should be integrated into the five content areas. (*Chapter 3*)
- In curriculum documentation, critical ideas in each content domain need to be explicated and expressed as narrative descriptors. These critical ideas, derived from learning paths, should serve as reference points for planning and assessment. In presenting these ideas, over-specification should be avoided. Learning outcomes arising from these also need to be articulated. (*Chapter 3*)
- Narrative descriptors of mathematical development, that is, descriptions of critical ideas, should be developed in class bands, e.g., two years. These critical ideas indicate shifts in children's mathematical reasoning in each of the content domains. (*Chapter 3*)
- The principles of equity and access should underpin the redeveloped mathematics curriculum. The nature of support that enables exceptional

children (those with developmental delays and those with exceptional talent), children in culturally diverse contexts and children in disadvantaged circumstances to experience rich and engaging mathematics should be specified. (*Chapter 4*)

- Additional support/intervention for children at risk of mathematical difficulties should begin at a much earlier point than is specified in the current guidelines. (*Chapter 4*)
- Learning outcomes in mathematics should be cross-referenced with other areas of learning and vice-versa, in order to facilitate integration across the curriculum. (*Chapter 2*, *Chapter 4*)
- Additional time allocated for mathematics should reflect the increased emphases on mathematization and its associated processes. Some of this additional time might result from integration of mathematics across areas of learning. While integration has the potential to develop deep mathematical understanding, the challenges that it poses to teachers must be recognised. (*Chapter 3*, *Chapter 4*)
- Ongoing communication and dialogue with parents and the wider community should focus on the importance of mathematics learning in the early years, the goals of the mathematics curriculum and ways in which children can be supported to achieve these goals. (*Chapter 5*)
- Structures should be put in place that encourage and enable the development of mathematical knowledge for pre-service and in-service teachers. Educators need to be informed about goals, learning paths and critical ideas. Records of practice, to be used as a basis for inquiry into children's mathematical learning and thinking, need to be developed. (*Chapter 6*)
- Educators need to be given opportunities to interrogate and negotiate the redeveloped curriculum with colleagues as it relates to their setting and

context. Time needs to be made available to educators to engage in collaborative practices such as lesson study. (*Chapter 6*)

- Given the complexities involved, it is imperative that all educators of children aged 3–8 years develop the knowledge, skills, and dispositions required to teach mathematics well. (*Chapter 6*)
- Given the central importance of mathematics learning in early childhood and as a foundation for later development, mathematics should be accorded a high priority, at both policy and school levels, similar to that accorded to literacy. (*Chapter 4*, *Chapter 5*)



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